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On the Application of the Parks-McClellan Algorithm to the Design of Quadrature Demodulation Filters (U)

R. Inkol and D.P. Truong Nguyen

DEFENCE RESEARCH ESTABLISHMENT OTTAWA

TECHNICAL REPORT
DREO TR 1999-063
June 1999

National Defence Défense nationale

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EW Signal Processing Group
Electronic Support Measures Section

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5BB12

ABSTRACT

New results are presented concerning the use of the Parks-McClellan algorithm to design filters for digital quadrature demodulators based on quadrature mixing and lowpass filtering concepts. The use of a 4:1 ratio between the sampling rate and intermediate frequency to reduce computational cost complicates this problem. Since the in-phase (I) and quadrature (Q) filters become odd and even length filters, respectively, the matching of the passband gains becomes an important error source. Consequently, the problem is to find the best design for a pair of filters rather than the best design for a single filter. One issue is whether to design the I and Q filters separately, or derive them from a prototype filter. Another concerns techniques for designing fractional-band filters if these are desired. The performance data presented in this paper shows that the quadrature demodulator accuracy has a complex dependence on the approach and specifications used to design the filters. Since good matching of the filter gains in the passband occurred only under certain conditions, significant performance losses can occur unless some care is taken in designing the filters.

RÉSUMÉ

Le présent rapport décrit de nouveaux résultats sur l'utilisation de l'algorithme de Parks-McClellan dans la conception de filtres pour démodulateur numériques en quadrature fondée sur les concepts de filtrage en quadrature passe-bas et mélangeur. L'utilisation du rapport 4:1 entre la fréquence d'échantillonnage et la fréquence intermédiaire pour la réduction du coût des calculs complique le problème. Puisque les filtres en phase (I) et en quadrature (Q) deviennent des filtres de longueur de parité pair et impair, l'adaptation des gains de la bande passante devient une importante source d'erreurs. Le problème est donc de concevoir le meilleur modèle d'une pair de filtres, et non pas de concevoir le meilleur modèle d'un seul filtre. Plusieurs préoccupations sont apparues au cours des travaux, notamment au sujet des filtres et des techniques à utiliser: il s'agissait de déterminer si les filtres en phase et ceux en quadrature devaient être conçus séparément ou s'ils devaient être produits à partir du filtre prototype; il a aussi été question de définir les techniques de conception des filtres de bande fractionnaire, si jamais on avait besoin. Les données relative à la performance présentées dans cet article révèlent que l'exactitude du démodulateur en quadrature dépend d'une manière complexe de l'approche et des spécifications utilisées dans la conception des filtres. Puisque le bon couplage de gain ne se produit que dans certaines conditions, d'importantes pertes de la performance peuvent survenir, à moins que la conception des filtres ne soit fait avec soin.

EXECUTIVE SUMMARY

Quadrature demodulation is useful in radar, communications and electronic warfare systems for obtaining complex baseband representations of real bandpass signals. Digital signal processing approaches for quadrature demodulation have significant advantages over their analog counterparts in stability and freedom from the error mechanisms generally found in analog circuits. The well-known quadrature mixing and lowpass filtering approach has some attractive features. The stopbands associated with the symmetric bandpass frequency response characteristic of the quadrature demodulator are often useful in practical applications. They can remove some of the quantization noise and spurious signals generated by the analog-to-digital converter and analog signal processing blocks preceding the quadrature demodulator. If linear phase finite impulse response (FIR) filters are used for the in-phase (I) and quadrature (Q) filters, then problems with nonlinear phase shifts are avoided. The computational cost of this approach is an issue, but it can be substantially reduced by:

- sampling the input signal at 4x its center frequency;
- reducing the sampling rate in intermediate processing stages to the lowest value consistent with the usable bandwidth;
- designing the in-phase filter to be a fractional-band filter so that the value of some of the filter coefficients is zero.

The frequency response matching of the I and Q filters is a critical issue if the generation of spurious signals is to be avoided. A given specification for frequency response matching can be achieved by making the passband frequency response ripple of the I and Q filters sufficiently small by increasing the transition bandwidth and/or the number of filter coefficients. However, the first of these reduces the bandwidth while the second increases implementation cost. If the filters can be designed so that their passband frequency response mismatch is small relative to the ripple of the individual filters, a useful reduction in the number of filter coefficients needed to provide adequate performance for a given application may be feasible.

The Parks-McClellan algorithm is widely used for designing FIR filters. It is an optimal design method in the sense that the maximum error relative to the desired frequency response specification is minimized. Furthermore, the ability to define arbitrary frequency response specifications and make tradeoffs between the pass and stopband frequency response errors is very useful. This paper presents the results of an investigation into the issues involved in the use of the Parks-McClellan algorithm to design the I and Q filters:

- whether to design separately the filters using a common frequency response specification or derive them from a single prototype filter;
- effects of the choice of frequency response specifications and number of filter coefficients;
- relative merits of different approaches for using the Parks-McClellan algorithm to design fractional-band filters.

The design problem is complicated by the various choices for the number of filter coefficients and the frequency response design specifications. For the Parks-McClellan algorithm,

these include the widths of the pass, transition and stopbands and the weights to be applied to the evaluation of errors in the pass and stopbands. In this study, the problem was simplified by designing the I and Q filters to a half-band frequency response specification (i.e., equal pass and stopband widths and weights). For each design approach six families of filters were constructed by varying the width of the transition band between the pass and stopbands. Each family of filters consisted of pairs of I and Q filters whose total number of coefficients was odd and ranged from 23 (e.g., I and Q filters constructed with 11 and 12 coefficients, respectively) to 223. The frequency domain behaviour of each pair of I and Q filters was computed and tabulated. Phase error bounds were used rather than other measures of frequency response mismatches to facilitate comparisons with previously published performance data.

Since the frequency response behaviour of single filters designed using the Parks-McClellan algorithm is well understood, this investigation placed emphasis on the frequency response matching of the I and Q filters. For comparison purposes, additional sets of I and Q filters were designed using the window design method. The Kaiser window was selected because of the availability of a design parameter, which allows the size of the transition band width to be specified. Also it is an optimal window in the sense that the energy in the frequency domain sidelobes is minimized.

Several important conclusions can be drawn from the observed results. First, the phase error, and other performance parameters affected by the matching of the I and Q filter frequency responses, are sensitive to the choice of design approach and can have a complex dependence on the number of filter coefficients. Second, the Parks-McClellan algorithm can be used to separately design I and Q filters which have very favourable properties. For example, for the design specifications considered in this report, very good matching of the I and Q filter magnitude responses was observed when the total number of filter coefficients, *M*, was chosen such that (*M*-1) or (*M*-7) are divisible by 8. A further important result is that virtually identical results can be achieved when the in-phase filter is designed using Vaidyanathan's technique to obtain a true half-band filter design where nearly half the filter coefficients are equal to zero. Consequently, a useful saving in computational cost can be obtained without any performance loss.

While the work reported here focuses on designs where the I filter has a half-band frequency response characteristic, we have shown that the results for other design specifications are similar in some respects. For example, when the Parks-McClellan algorithm was used to separately design I and Q filters having other passband widths, the dependence of the phase error performance on the number of filter coefficients differed in detail, but favourable choices for the number of filter coefficients could still be identified. These results provide useful insights into the I and Q filter design optimization problem and can be used to identify possible design choices given known application performance requirements.

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1. INTRODUCTION

The processing of a real bandpass signal nominally centered on an intermediate frequency, f_{IF}, to form an in-phase (I) and quadrature (Q) signal representation is useful in coherent radar, communication and electronic warfare systems. The classical approach for obtaining I and Q signals from an analog bandpass signal involves quadrature mixing and lowpass filtering as shown in Figure 1. If the lowpass filters each have a bandwidth, B, then the quadrature demodulator has a symmetric bandpass magnitude response with a bandwidth $2B$ centered on $f_{IF}=2\pi\omega_c$, the frequency of the local oscillator.

Analog quadrature demodulator implementations are known to have problems with the errors resulting from amplitude and phase mismatches between the I and Q channels [1]-[2]. These can be controlled by careful component matching and/or performing error compensation in subsequent post-processing. However, such an approach can incur significant economic costs. Digital approaches for performing quadrature demodulation on a sampled and digitized bandpass signal have potential advantages in these respects, but their computational cost is often an issue. This problem has motivated much work on the development of efficient algorithms [3]-[5].

A very important idea for reducing computational cost involves setting the sampling rate to 4 times the intermediate frequency, f_{IF}, of the input signal. By applying this constraint to the digital equivalent of the quadrature mixing and lowpass filtering approach shown in Figure 1, the mixing signals can be formed from the trivial sequence $\{0, 1, 0, -1, 0, \ldots\}$. Second, the product signals resulting from the mixing operations can be decimated by 2 with no loss of information. Finally, the I and Q filters can be constructed by sub-sampling a finite impulse response (FIR) lowpass filter defined by the $M=2K+1$ filter coefficients

$$h_k \in \{h_{-K}, \ldots, h_{-3}, h_{-2}, h_{-1}, h_0, h_1, h_2, h_3, \ldots, h_K\}. \tag{1}$$

The resulting I and Q filters have an odd and even number of filter coefficients, respectively, defined by

$$\begin{aligned} h_k^I &\in \{\ldots, h_{-k}, \ldots, h_{-2}, h_0, h_2, \ldots, h_k, \ldots\} \\ &\qquad \text{for even } k, \\ h_k^Q &\in \{\ldots, h_{-k}, \ldots, h_{-3}, h_{-1}, h_1, h_3, \ldots, h_k, \ldots\} \\ &\qquad \text{for odd } k. \end{aligned} \tag{2}$$

Note that the two filters require a total of M coefficients instead of $2M$ coefficients. and that the sampling rate of the data processed in one channel is $f_s'=f_s/2$ instead of f_s. Figure 2 shows a practical implementation of a digital quadrature demodulator based on these ideas. Note that each of the resulting lowpass filters has a passband gain half that of the prototype filter.

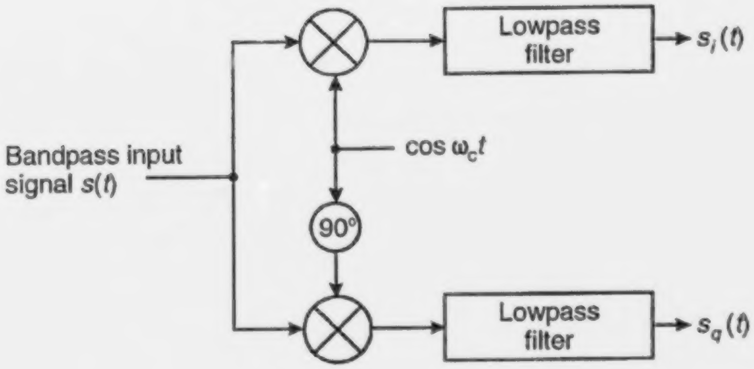
Lowpass filter
$s_i(t)$
$\cos \omega_c t$
Bandpass input signal $s(t)$
90°
Lowpass filter
$s_q(t)$

Figure 1. Analog quadrature demodulator using quadrature mixing and lowpass filtering.

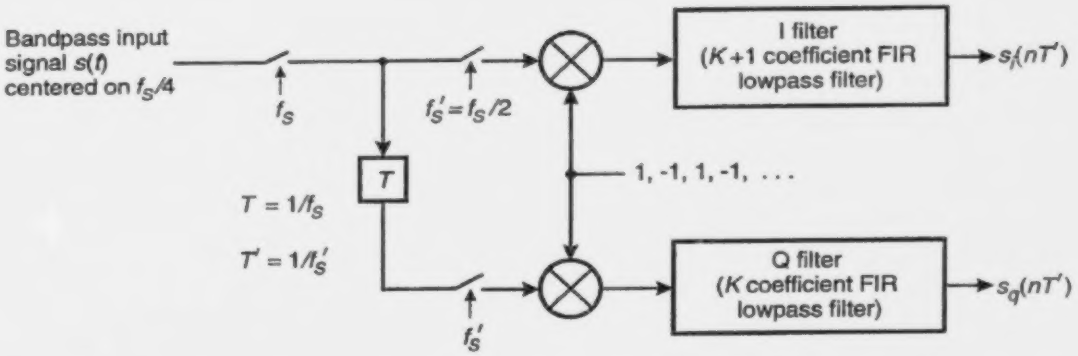
Bandpass input signal $s(t)$ centered on $f_S/4$
f_S
$f_S' = f_S/2$
T
$T = 1/f_S$
$T' = 1/f_S'$
f_S'
1, -1, 1, -1, . . .
I filter (K+1 coefficient FIR lowpass filter)
$s_i(nT')$
Q filter (K coefficient FIR lowpass filter)
$s_q(nT')$

Figure 2. Efficient quadrature demodulator having I and Q filters with a total of M=2K+1 filter coefficients where the I filter has one more filter coefficient than the Q filter (i.e., (M-1) divisible by 4). If the Q filter has one more coefficient than the I filter (i.e., (M-3) divisible by 4), delay T must be moved to the input of the I filter.

Further reductions in computational cost occur if the prototype filter is designed to be a fractional band filter [5]. The coefficients of a N_{th} band FIR filter with M coefficients satisfy the constraints [6]

$$h_0^N = 1/N \quad and \quad h_{\pm N} = h_{\pm 2N} = \ldots = h_{\pm mN} = 0, \qquad m = N \text{ int } (M-1)/2N, \tag{3}$$

where int denotes "the integer value of". Consequently, a saving of nearly $1/N$ in computational cost can be obtained for a given value of M. For the $N=2$ case, the prototype filter is a half-band filter and the I filter simplifies to a delay. This case is known to be exactly equivalent to the Hilbert transformer approach [5].

A recent idea for reducing computational cost is to design the prototype filter from which the I and Q filters are formed to be a quarter-band filter [5]. Since the filter coefficients of a quarter-band filter satisfy the constraints embodied in (3), the computational cost is reduced by nearly one quarter. Note that the I filter derived from the quarter-band prototype filter is a half-band filter. Half-band filters have symmetrical pass and stopbands with equal ripple [6]. Consequently, the quadrature demodulator has a -6 dB bandwidth of ~ $f_s/4$ centred on f_{IF}. For a prototype filter having a transition bandwidth, *TBW*, normalized to 1 at the Nyquist frequency (i.e., half of the sampling rate), the I and Q filters each have a transition band width 2*TBW* and the pass, transition and stopbands of the resulting quadrature demodulator are as specified in Table 1. Since the maximum passband width is always less than $f_s/4$, a further reduction of the sampling rate at the filter outputs to $f_s/4$ can be obtained by performing a second decimation by 2 to reduce the output data rate for each of the I and Q signals to $f_s/4$. As pointed out in [5], the inclusion of this decimation stage allows an equivalent implementation without explicit mixing, the high pass filter digital quadrature demodulator, to be constructed. Note that the resultant alias-free bandwidth is [$f_s/8$, $3f_s/8$] and that spectral components outside this bandwidth and which remain after the filtering, will alias into it if the second decimation by 2 is performed.

Table 1. Specifications of passband, transition band, and stopband for proposed quadrature demodulator (quarter-band prototype filter).

	bandwidth specifications (normalized to $f_s/2$)
passband	[0.25+*TBW*/2, 0.75-*TBW*/2]
transition bands	[0.25-*TBW*/2, 0.25+*TBW*/2], [0.75-*TBW*/2, 0.75+*TBW*/2]
stopbands	[0, 0.25-*TBW*/2], [0.75+*TBW*/2, 1]

In comparison with the Hilbert transform approach, the proposed approach has the important difference of providing a bandpass magnitude response characteristic. In practical applications, the stopbands in the quadrature demodulator magnitude response are often very useful for suppressing undesired signals:

- DC offsets introduced by the analog-to-digital converter (ADC);
- spurious signals (e.g., harmonic distortion) introduced by the ADC or other sources;
- quantization noise (~3 dB reduction, equivalent to improving resolution by ~½ bit, results from the choice of the half-band magnitude response specification).

Furthermore, the additional filtering can often relax the design specifications for the analog intermediate frequency (IF) filters preceding the ADC.[1] Consequently, the proposed quadrature demodulator should be viewed as a combination of digital IF filter and quadrature demodulator when evaluating system design issues and trade-offs.

The performance achievable with the proposed design approach was investigated in [7], but this work concentrated on the use of window methods for designing the prototype filter from which the I and Q filters were derived. This paper extends the previous investigation to address approaches for using the Parks-McClellan algorithm [8] to design the I and Q filters.

---

[1] This is important in applications where linear phase characteristics are desired; analog filters designed to have good phase linearity often have relatively poor selectivity and digital bandpass filtering may be required to provide the required selectivity. Also, in multi-channel systems where tracking of the delay or phase of several nominally identical channels is required, the replacement of analog signal processing by digital techniques is often advantageous.

2. DESIGN CONSIDERATIONS FOR QUADRATURE DEMODULATION FILTERS

A common objective concerning the design of digital filters is to find a design that meets performance criteria associated with the target application with the minimum number of filter coefficients (i.e., obtain the required performance at the lowest computational cost). In typical applications where digital filters are used to process real signals, the performance measures of interest for the magnitude response include:

a) widths of the pass, transition and stopbands;
b) passband ripple;
c) stopband attenuation.

In the digital quadrature demodulator considered in this paper, an input signal is separately processed in two channels. The digital filters in these channels differ in the number of coefficients and have magnitude responses which do not exactly match, even if the filters are designed to a common frequency response specification. One interpretation of the effects of a mismatch in the passband frequency magnitude responses involves the introduction of spurious phase and amplitude modulations. For a sinusoidal signal whose frequency is offset from the center input frequency of the quadrature demodulator by f_i, the peak phase error in radians is given by

$$\varphi_e(f_i) = \arctan(R(f_i)) - \pi/4, \tag{4}$$

where $R(f_i)$ is the ratio of the gains of the I and Q filters [2]. Similarly the ratio of the peak-to-peak amplitude of the amplitude modulation, $\Delta A(f_i)$, to the mean value of the envelope of the sinusoidal signal, $\bar{A}(f_i)$, is given by

$$\frac{\Delta A(f_i)}{\bar{A}(f_i)} = \frac{2\,|1-R(f_i)^2|}{(1+R(f_i)^2)}. \tag{5}$$

Another interpretation of the effects of a mismatch in the magnitude responses concerns the generation of a spurious signal at the image frequency of a sinusoidal input signal. A theoretical result for the image rejection ratio (I_{rr}) in decibels is given by [1]

$$I_{rr}(f_i) = 10\,Log\frac{1+2R(f_i)+R(f_i)^2}{1-2R(f_i)+R(f_i)^2}. \tag{6}$$

Note that these results are dependent on the assumption that there is no relative phase error between the I and Q filters. This assumption can be satisfied by FIR filters whose impulse responses are symmetric, but is not valid for infinite impulse response (IIR) filters.

In many practical applications, these errors are very undesirable. Consequently, the problem of designing and specifying a pair of quadrature demodulation filters is more complex than for a single filter since the performance of the quadrature demodulator is dependent on the magnitude responses of both filters. Desirable design goals for the passband band magnitude response specifications of quadrature demodulation filters can be identified:

(a) minimization of the maximum passband ripple of the I and Q filters;
(b) close matching of passband widths;
(c) close matching of passband magnitude responses.

Note that the best case result for (c) occurs if the passband magnitude responses of the I and Q filters are identical. In practice, the mismatches between the magnitude responses can be very important, even if the filters were derived from a common prototype filter as described in Section 1. Also, previously reported research has shown that significant differences can result from the choice of filter design method [7].

Some insights into useful design specifications can be inferred from a physical interpretation of the magnitude responses of a pair of I and Q filters shown in Figure 3. The passband magnitude responses of the individual I and Q filters are conventionally specified by the passband widths, $F_{p,I}$ and $F_{p,Q}$, and the peak-to-peak ripple measurements of their magnitude responses, $2\delta_I$ and $2\delta_Q$. The quadrature demodulator has a symmetric bandpass magnitude response for input signals; this can be obtained by folding the magnitude responses of the lowpass I and Q filters about $f_s/4$. The two magnitude responses define bounds for the gain of the quadrature demodulator. When a sinusoidal input signal is applied, the instantaneous gain is dependent on the instantaneous phase of the signal and will oscillate between the values of the two bounds at a rate determined by the frequency offset of the signal from the center frequency of the quadrature demodulator, $f_s/4$. Consequently, the difference between the upper and lower bounds of the quadrature demodulator passband gain observed over the passband, the peak-to-peak ripple, $2\delta_{IQ}$, is a measure of the range of passband quadrature demodulator gains. It is measured over the composite passband defined by $F_{p,IQ}$ which is bounded by $F_{p,I}$ and $F_{p,Q}$. Note that $2\delta_{IQ}$ is never less than the largest passband peak-to-peak ripple of the two filters and can be much larger, particularly if the gains of the filters differ significantly. Sometimes, $2\delta_{IQ}$ can be improved by modifying the gain of one filter by appropriately scaling its filter coefficients. The peak magnitude of the mismatch in the passband magnitude responses of the filters is defined by Δ_{IQ}. Note that $(1-\Delta_{IQ})$ is approximately equal to the ratio of magnitudes, $R(f_i)$, used in (4)-(6) if Δ_{IQ} is small. Alternative means of indirectly measuring of Δ_{IQ} are provided by the worst case image rejection ratio or peak phase error defined by (6) and (4), respectively. Since these performance measures include the effects of other error sources, which can be important for some quadrature demodulator designs, they have the advantage of facilitating straightforward performance comparisons. The magnitude response specifications can also be defined using mean square instead of peak or peak-to-peak measurements, but this performance measure has the limitation that it cannot ensure the detection of unacceptable peak errors when they exist over a small frequency range.

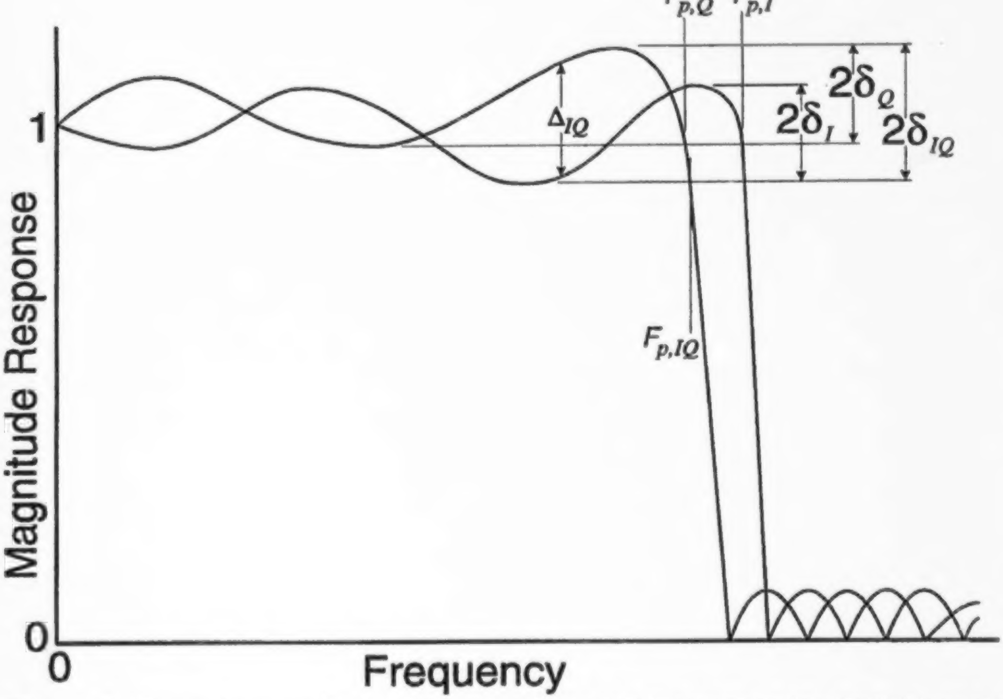

Figure 3. Example magnitude responses of I and Q filters showing definitions of passband specifications for bandwidth, $F_{p,I}$, $F_{p,Q}$ and $F_{p,IQ}$, peak-to-peak ripple, $2\delta_I$, $2\delta_Q$, and $2\delta_{IQ}$, and magnitude response mismatch, Δ_{IQ}.

3. DESIGN METHODS BASED ON THE PARKS-McCLELLAN ALGORITHM

The Parks-McClellan algorithm [8] is an optimal design method in the sense that it minimizes the maximum absolute error in the magnitude response for the chosen pass and stopband specifications. Therefore, it can achieve a given performance specification in the minimax sense with fewer filter coefficients than are required if window design methods are used. Another advantage concerns the flexibility provided by the capability for arbitrarily defining pass and stopbands and the weights with which the errors of their magnitude responses are considered. Consequently, this algorithm has received wide acceptance for use in designing FIR filters and is commonly implemented in commercial mathematics software such as MATLAB™.

This paper focuses on quadrature demodulator designs in which the I filters are half-band filters. A practical problem with the use of the Parks-McClellan algorithm concerns the design of quarter and half-band filters where (3) applies. When a window design method is used, fractional-band filters can be obtained directly by simply selecting an appropriate bandwidth specification. However, this condition is not sufficient with the Parks-McClellan algorithm. Several possible design methods based on the Parks-McClellan algorithm have been proposed, but these differ significantly.

3.1 Methods for Designing I and Q Filters

There are two general approaches for designing I and Q filters using a given filter design method:

A. *Prototype filter design method.* As noted in section 1, the application of the constraint $f_{IF}=f_s/4$ permits the construction of the in-phase and quadrature filters, respectively, from the even and odd coefficients of a prototype filter. Since this procedure implies that the I and Q filter impulse responses are decimated by 2 versions of the prototype filter impulse response, the prototype filter must be designed for pass and transition band widths which are half those desired for the I and Q filters. This paper considers the use of prototype filters which are designed according to the specification

$$f_1 + f_2 = 0.5 \quad and \quad W_1 = W_2, \tag{7}$$

where f_1 and f_2 are the passband and stopband cutoff frequencies, respectively, and W_1 and W_2 are the weights to be given to the pass and stopband errors, respectively. This is the magnitude response specification of a quarter-band filter and the derived I and Q filters have magnitude responses characteristic of half-band filters.[2] Although the designs obtained by this procedure do not satisfy exactly (3), they are useful for comparison with designs which do. Note that the frequencies are normalized to the Nyquist frequency.

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[2] Note that passband gains of the I and Q filters formed by sub-sampling the prototype filter will be approximately half that of the prototype filter. This can be corrected, if desired, by scaling the filter coefficients by a factor of 2.

B. *Separate filter design method.* The I and Q filters can be designed separately with a common magnitude response specification. Since we are interested in designs which result in the in-phase filter being a half-band filter, we used a design specification characteristic of half-band filters

$$f_1 + f_2 = 1.0 \quad and \quad W_1 = W_2. \tag{8}$$

Although the I filter does not have coefficients with values exactly equal to zero, the performance results obtained using these filters are useful for comparison with design approaches which result in the I filter being a true half-band filter. Note that other choices can be made for the pass and stopbands and their weights to meet specific application requirements. However, if this is done, the performance results will differ from those presented in this paper.

3.2 Design Methods for Fractional-Band Filters

The use of a half-band filter for the I filter is attractive because of the saving in computational cost. If the I and Q filters are designed separately, a procedure for directly designing a half-band filter is required. However, if the I and Q filters are derived from a prototype filter, the prototype filter must be a quarter-band filter. The Parks-McClellan algorithm cannot be used to obtain directly Nth band filters where $(M\text{-}3)/N$ coefficients have values of zero. However, several procedures which make use of a priori information for designing fractional-band filters using the Parks-McClellan algorithm have been proposed:

A. *Mintzer technique.* A procedure has been proposed by Mintzer [6] for designing fractional-band FIR filters using the Parks-McClellan algorithm. First, a filter with the desired number of coefficients and the appropriate magnitude response specification is designed using the Parks-McClellan algorithm. For half and quarter-band filters, the specifications given in (8) and (7), respectively, are suitable. Second, the constraints defined in (3) are applied to the filter coefficients. Filters designed using this procedure are known to be sub-optimal, but this is often considered to be an acceptable trade-off for ease of design.

B. *Vaidyanathan/Estola techniques.* A more sophisticated way of designing half-band filters has been proposed in [9] by Vaidyanathan. First, the Parks-McClellan algorithm is used to design a one-band filter with $(M+1)/2$ coefficients, a passband width of $2F_p$, transition band width of 1.0-$2F_p$ and a stopband of zero width at 1.0. Second, the full-band filter impulse response is upsampled to form an M coefficient filter by inserting a single zero-valued coefficient between each pair of coefficients after which the center coefficient is set to ½. The Parks-McClellan algorithm is well suited for designing the full band filters since the widths and weights of the pass and transition bands can be accurately set. This idea has been extended in [10] to define a recursive method of constructing Nth band filters where N is a power of 2. For example, a quarter-band filter can be formed by interleaving the coefficients of a half-band filter and an even length filter which has the same magnitude response specification. However, this idea does not have independent significance for designing a quarter-band prototype filter from which the I and Q filters will be derived. Since they would be constructed by reversing the process of forming the quarter-band filter from the coefficients of two separately designed filters, the result would be the same as if the I and Q filters were separately designed using the Parks-McClellan algorithm and Vaidyanathan's technique.

C. *Saramaki technique*. A more general approach has been described in [11] for designing *N*th band filters. However, it is relatively complex to implement and is not considered further in this report.

Consequently, this paper considers only two approaches for designing prototype filters. The first involves the direct application of the Parks-McClellan algorithm using the quarter-band frequency response specification given in (7).[3] The second involves the Mintzer procedure where the constraints defined in (3) are applied directly to the designs obtained with the first method.

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[3] Note that the first method does not provide the potential saving in computational cost which results if (3) is satisfied.

4. EXPERIMENTAL APPROACH

The design and evaluation of families of filter designs obtained with the proposed design approaches was carried out in the MATLAB™ 5.3 programming environment. Using the MATLAB™ remez function to implement the McClellan-Parks algorithm, families of I and Q filters were constructed with the design approaches discussed in Section 3. Each family of filters consisted of pairs of I and Q filters whose total number of coefficients was odd and ranged from 23 (e.g., I and Q filters constructed with 11 and 12 coefficients, respectively) to 223. For large transition band widths, the maximum number of filter coefficients had to be reduced to avoid convergence problems. The filters were designed using a common frequency response specification consistent with (7) or (8) as required. Six families of filters were designed using each design method. Transition band widths, $TBW \in \{0.075, 0.100, 0.150, 0.200, 0.250, 0.300\}$, were directly used in the construction of the prototype filters.[4] To obtain transition band widths equal to those of the I and Q filters derived from the prototype filters, the transition band widths used to separately design I and Q filters had to be doubled. For consistency, all transition band widths given in this document correspond to the prototype filter design method and are normalized to 1 at the Nyquist frequency (i.e., half the sampling rate at the filter input).

The MATLAB™ freqz function was used to obtain the magnitude frequency responses for each pair of I and Q filters at 512 discrete frequencies uniformly distributed over the range $[0, f_s'/2]$. From this data, the actual passband widths $F_{p,I}$, $F_{p,Q}$ and $F_{p,IQ}$, and magnitude response errors, $2\delta_I$, $2\delta_Q$, $2\delta_{IQ}$ and Δ_{IQ} were computed. Also, the phase error bound was computed using (4) and the magnitude of the peak phase error was obtained for the measured passband width $\Delta F_{p,IQ}$. Phase error bounds were used rather than other measures of magnitude response mismatch to facilitate comparisons with previously published performance data and because they are of direct importance in applications such as the demodulation of digital signals. The phase error bound data was examined to investigate the nature of its dependence on the filter design parameters. Also, for selected cases, the passband magnitude responses of the I and Q filters were computed and plotted.

As a check to ensure the validity of using (4), the image rejection ratio was measured directly for selected cases. This was done by simulating the behaviour of the quadrature demodulator for a sinusoidal input signal at various frequencies within the quadrature demodulator passband width. The powers contained in the signal and negative image components were computed from the power spectrum of the I and Q signals obtained by using a 512 point complex FFT. A window was not used with the FFT since problems with spectral leakage were avoided by selecting the frequencies of the sinusoidal signals to correspond to the FFT analysis frequencies. In all of the cases selected, the image rejection ratio results were consistent with the phase error bound data.

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[4] Note that the larger transition bandwidths would be unsuitable for most applications as they substantially reduce the passband width relative to the sampling rate.

5. PERFORMANCE RESULTS AND ANALYSIS

5.1 Results

Figures 4-7 plot the measured passband peak phase error data for families of quadrature demodulator designs which were obtained using the four filter design methods. Example passband magnitude responses for $M \in \{81, 83, 85, 87\}$ are shown for these design methods in Figures 8-11. Figure 12 plots, the peak-to-peak passband magnitude response error, $2\delta_{IQ}$, as a function of the total number of filter coefficients, M, for pairs of I and Q filters, which were separately designed using the Parks-McClellan algorithm. Additional performance data for the measured passband widths $F_{p,I}$, $F_{p,Q}$, and $F_{p,IQ}$, and the peak-to-peak magnitude response passband ripple measurements, $2\delta_I$, $2\delta_Q$ and $2\delta_{IQ}$ are provided in Appendicies A-D.

5.2 Discussion and Comparison of Design Approaches

Figures 4-7 show an overall downward trend for the peak phase error bound data as the number of filter coefficients and/or the transition bandwidth are increased. This largely reflects the expected decline in the ripple in the passband magnitude response. Nevertheless, significant performance differences between the design approaches can be observed:

5.2.1. Separate Filter Design Methods

A.1. *Parks-McClellan*. The phase error bound results were usually very good when (M-1) or (M-7) were divisible by 8 and inferior for the other cases, particularly when (M-5) was divisible by 8. This periodic behaviour is very significant since the difference in the performance can vary by an order of magnitude for small changes in the number of filter coefficients. An examination of typical magnitude responses, such as those plotted in Figure 8 and the dependence of the peak-to-peak passband magnitude response error, $2\delta_{IQ}$, on the number of filter coefficients plotted in Figure 12, shows that the magnitude response ripple of the I and Q filters is very similar in the favourable cases with the result that $\delta_I \approx \delta_Q >> \Delta_{IQ}$. The relationship between these performance measures and the number of coefficients in each filter can be summarized as shown in Table 2. The cases for (M-1) and (M-7) divisible by 8 are of special interest. For (M-1) divisible by 8, the I filter has one more coefficient than the Q filter. However, the end coefficients of the I filter have values near zero as a result of the choice of (7) for the design specification and contribute very little to the behaviour of the filter. In this case, reducing the number of coefficients in the I filter by 2 has little effect, and, since the Q filter is unchanged, the overall result differs little. Consequently, the (M-1) case is of little independent significance.

These designs had desirable characteristics in other respects. As noted in Section 3, the passband mgnitude responses are known to be optimal in the minimax sense. Since the mean values of the passband gains are very similar, the matching of the filter gains over the passband cannot be significantly improved by modifying the relative gains of the filters. The passband and transition band widths of the I and Q filters are identical and conform with the design specification. This implies that the quadrature demodulator will have a bandpass magnitude response whose transition bands will correspond to the design specifications of the individual lowpass filters. Consequently, the quadrature demodulator designs based on these filters can be considered to be near-optimal. The one significant weakness is that the magnitude responses match poorly in the transition band [7].

Table 2. Relationship between total number of filter coefficients and characteristics of the I and Q filters.

Case	Lengths of I and Q filters	Values of end filter coefficients for I filter	Relative Passband Magnitude Response Ripple
(M-7) divisible by 8	K, K+1	~zero	$\delta_Q \approx \delta_I$
(M-5) divisible by 8	K+1, K	non-zero	$\delta_Q < \delta_I$
(M-3) divisible by 8	K, K+1	non-zero	$\delta_Q > \delta_I$
(M-1) divisible by 8	K+1, K	~ zero	$\delta_Q \approx \delta_I$

A.2. *Parks-McClellan/Vaidyanathan technique*. Very similar results were obtained when the Vaidyanathan technique was used to design half-band in-phase filters. One small difference was that the results for (M-1) and (M-7) were identical. This is to be expected; the non-trivial coefficients of the I filters for the two cases are identical since both filters were derived from the same even length filter design.[5] These results are important since they show that half-band I filters which exactly satisfy (3), and provide a useful saving in computational cost, can be designed using the Parks-McClellan algorithm without any performance degradation.

5.2.2. Prototype Filter Design Methods

B.1. *Parks-McClellan*. The results obtained for the passband magnitude response ripple and matching when the I and Q filters were derived from a prototype filter which was directly designed using the Parks-McClellan algorithm were generally inferior in comparison with the favourable cases of the separately designed I and Q filters. The periodic dependence on the number of filter coefficients was much weaker.

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[5] Note that this is not a general result whenever the I filter is a half-band filter. Significant differences can occur for some design methods. For example, if a window design method is used, the shape of the window is dependent on the number of coefficients (if the case of a rectangular window is neglected) and this can have significant performance implications though the number of non-zero filter coefficients does not change.

B.2. *Parks-McClellan/Mintzer technique*. The filter designs considered here differed from those in B.1 only in that some filter coefficients were modified by applying the constraint in (3). The major effect of this was to introduce some variability in the dependence of the measured phase error on the number of filter coefficients.

5.3 Comparison of Designs Obtained Using Parks-McClellan and Window Design Methods

Design approaches for FIR filters based on the use of window functions have limited flexibility, but offer the attraction of simplicity. In particular, fractional-band filters can be simply designed by appropriately selecting the filter bandwidth. For comparison with the results obtained with the Parks-McClellan algorithm, we employed the Kaiser window. This is one of the most attractive window functions for designing FIR filters [12]. It is an approximately optimal solution to the problem of designing a window function for a frequency response which minimizes the energy in the stopband. The window parameter, β, allows tradeoffs to be made between the transition bandwidth and the passband ripple and stopband attenuation. β is related to the amplitude of the stopband sidelobes in dB, A, and transition bandwidth, TBW, for a given number of filter coefficients, M, by

$$\beta = \begin{cases} 0.1102\,(A-8.7), & A>50 \\ 0.5842\,(A-21)^{0.4} + 0.07886\,(A-21), & 21<A<50 \\ 0, & A<21 \end{cases} \qquad \textbf{(9)}$$

and

$$A = 14.36\,M\,TBW + 7.95. \qquad \textbf{(10)}$$

A simple expression giving the value of β for a specific choice of TBW and M can be obtained by substituting (10) into (9). Note that TBW in (10) is normalized to the Nyquist frequency so as to be consistent with the implementation of the Parks-McClellan algorithm in MATLAB™.

5.3.1 Performance Results - I and Q Filters Separately Designed Using the Kaiser Window

The MATLAB™ fir1 function and equations (9) and (10) were used to design families of I and Q filters to a half-band filter frequency response specification for the same transition band widths used in the design of the corresponding Parks-McClellan filters. Note that this design approach results in slightly different values of β being used to design each pair of I and Q filters.[6] Figure 13 shows the peak phase error obtained for the families of I and Q filters as a function of the number of filter coefficients. More complete performance data is given in Appendix E. These results differ from those obtained when the Parks-McClellan algorithm was used to separately design the I and Q filters in that the periodic dependence on *M* is considerably weaker, although significant locally optimal points can still be identified for some smaller values of *M*. The passband magnitude response matching was inferior to that obtained when the Parks-McClellan algorithm was used when the number of filter coefficients was divisible by (*M*-7) or (*M*-1).

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[6] If the I and Q filters were designed using the same values of β for both filters, the results were usually slightly inferior.

5.3.2 Performance Results - I and Q Filters Derived From a Prototype Filter Designed Using the Kaiser Window

The MATLAB™ fir1 function was used to design families of prototype filters. Equations (9) and (10) were used to obtain values of β required to yield the same transition band widths as those previously used in the design of the filters by the Parks-McClellan algorithm. Figure 14 plots the peak phase error as a function of *M* for the families of I and Q filters. Example passband magnitude responses are plotted for $M \in \{81, 83, 85, 87\}$ in Figure 15. More complete performance data are given in the tables in Appendix F. For the smaller transition band widths, the general dependence of the phase error bounds on *M* resembled that observed when the Parks-McClellan algorithm was used to separately design the I and Q filters, but the periodicity was weaker. One result of interest is that the dependence of the phase error on *M* usually featured a well defined locally optimal value when (*M*-7) was divisible by 8. Since this case differs from the (*M*-1) divisible by 8 case only in that the window function applied to the non-zero filter coefficients is slightly different, it can be considered as a special case of (*M*-1) divisible by 8 with a slightly modified window function.

5.4 Overall Comparison of Parks-McClellan and Window Design Methods

The various design approaches were compared with respect to the passband phase error bounds and other performance parameters.

5.4.1 Passband Magnitude Response Error

The best performance results for peak-to-peak passband ripple, $2\delta_I$, $2\delta_Q$ and $2\delta_{IQ}$ are obtained when the I and Q filters are separately designed using the Parks-McClellan algorithm. This is to be expected since the Parks-McClellan algorithm is known to be an optimal design method in the minimax sense. The results obtained when the Parks-McClellan algorithm is used to design a prototype filter from which the I and Q filters are derived are inferior. The process of sub-sampling sets of filter coefficients designed using the Parks-McClellan algorithm results in sub-optimal filter designs. When the algorithm is used to design a single filter, the resulting filter coefficients have been optimized to minimize the peak magnitude response error. This results in an equiripple magnitude response across each of the pass and stopbands. However, if the filter coefficients are sub-sampled to form the I and Q filters, the resultant aliasing of the stopband ripple into the passband causes the equiripple behaviour to be lost, and the magnitude response errors are larger than if the filters were directly designed using the Parks-McClellan algorithm.

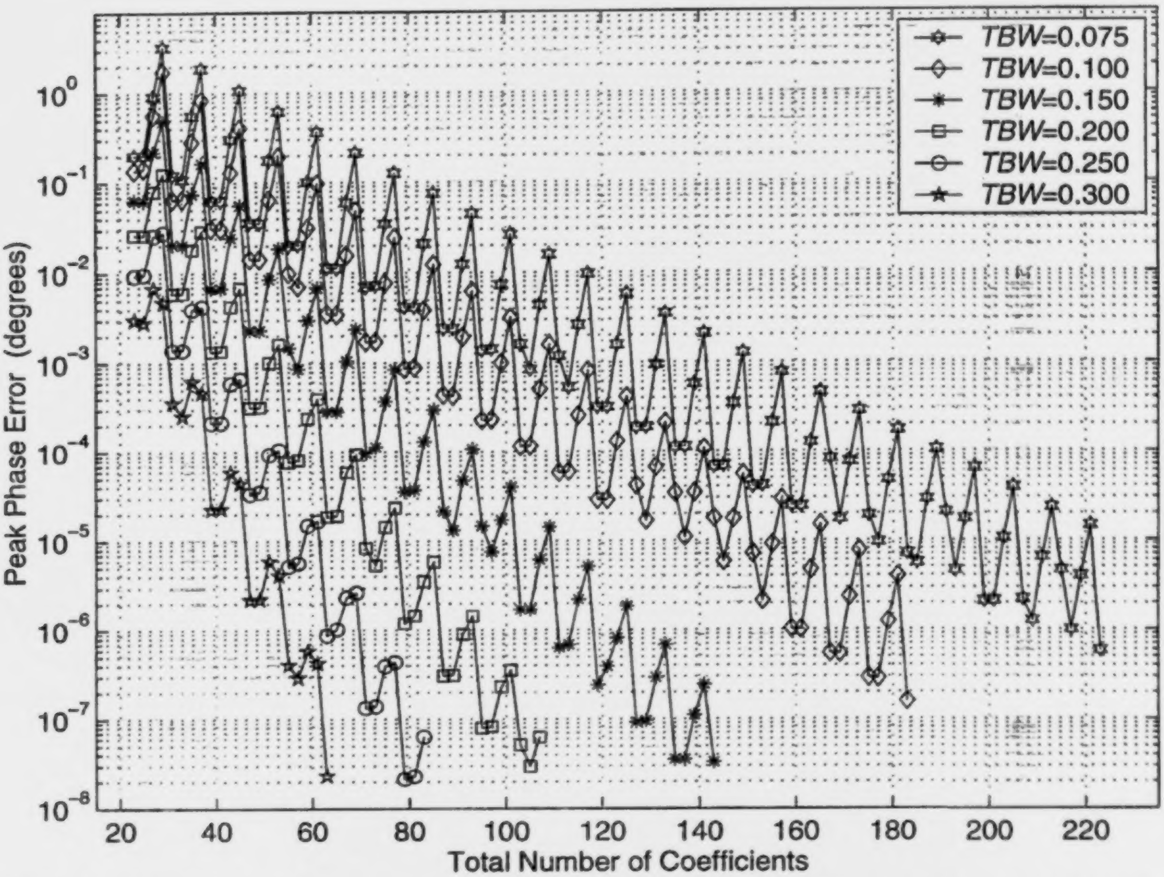

Figure 4. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of in-phase and quadrature filters. The filters were separately designed using the Parks-McClellan algorithm. Each of the transition band widths, *TBW*, is normalized to the Nyquist frequency and corresponds to the transition band width of a prototype filter from which I and Q filters having the same magnitude response specifications could be derived (i.e., the actual values of *TBW* used to design the filters are larger by a factor of 2).

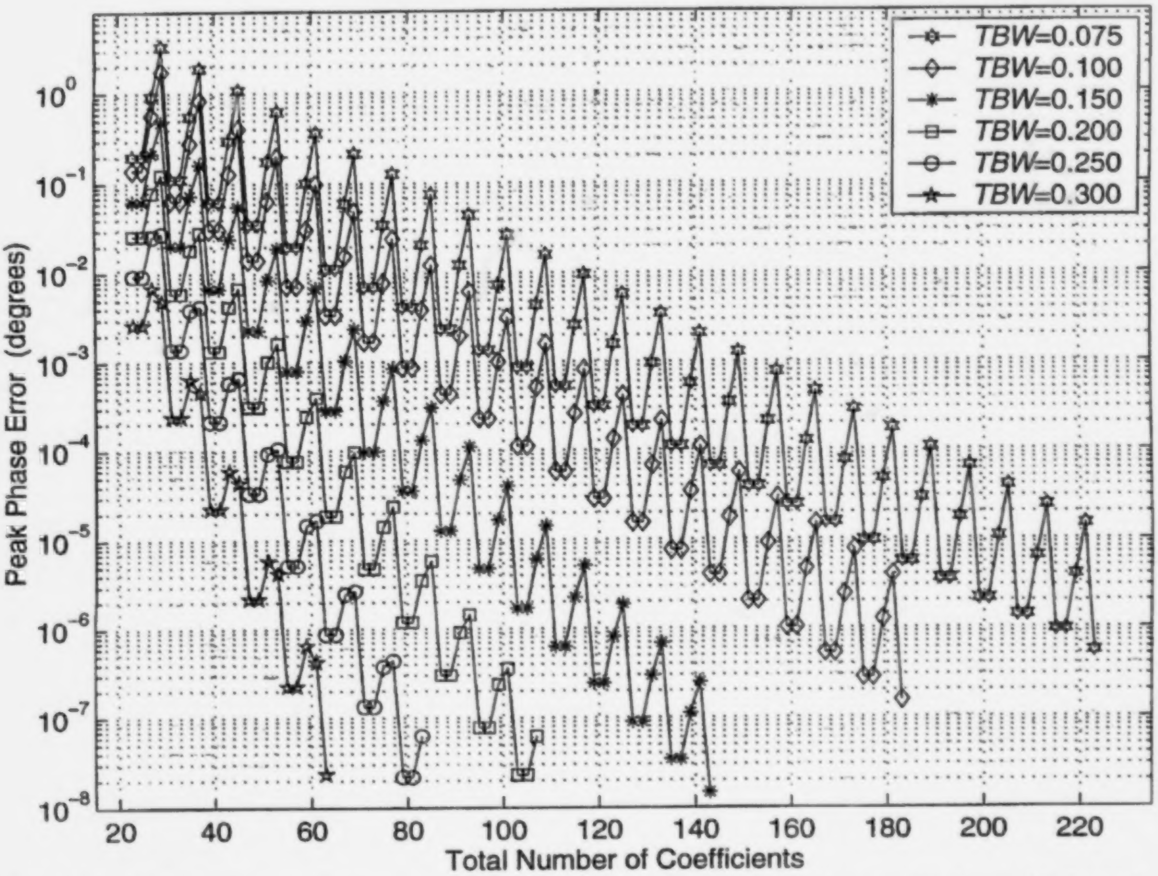

Figure 5. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of in-phase and quadrature filters. The filters were separately designed using the Parks-McClellan algorithm. Vaidyanathan's technique was used to design the in-phase filter to be a half-band filter. Each of the transition band widths, *TBW*, is normalized to the Nyquist frequency and corresponds to the transition band width of a prototype filter from which I and Q filters having the same magnitude response specifications could be derived.

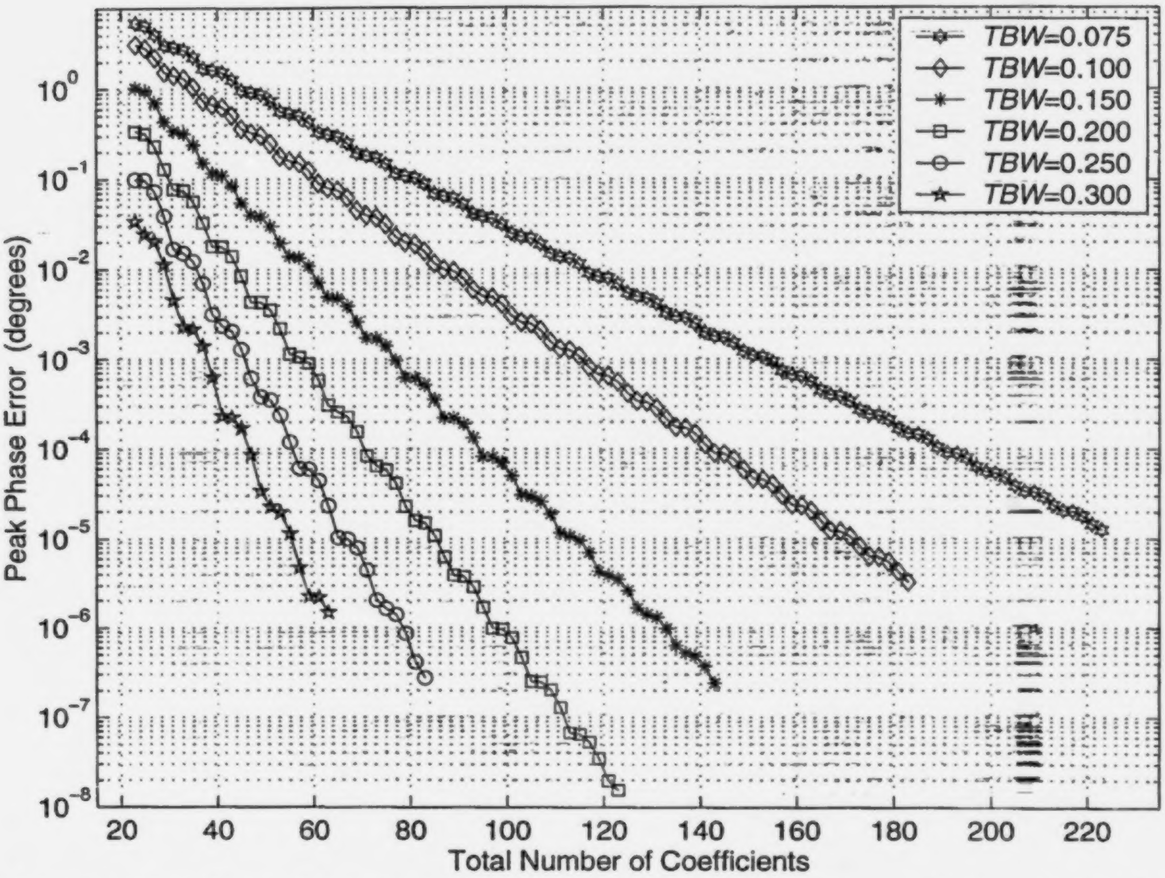

Figure 6. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of in-phase and quadrature filters. Each pair of I and Q filters was derived by sub-sampling the coefficients of a prototype filter designed using the Parks-McClellan algorithm. The transition band widths used in the design of the prototype filters, *TBW*, are normalized to the Nyquist frequency.

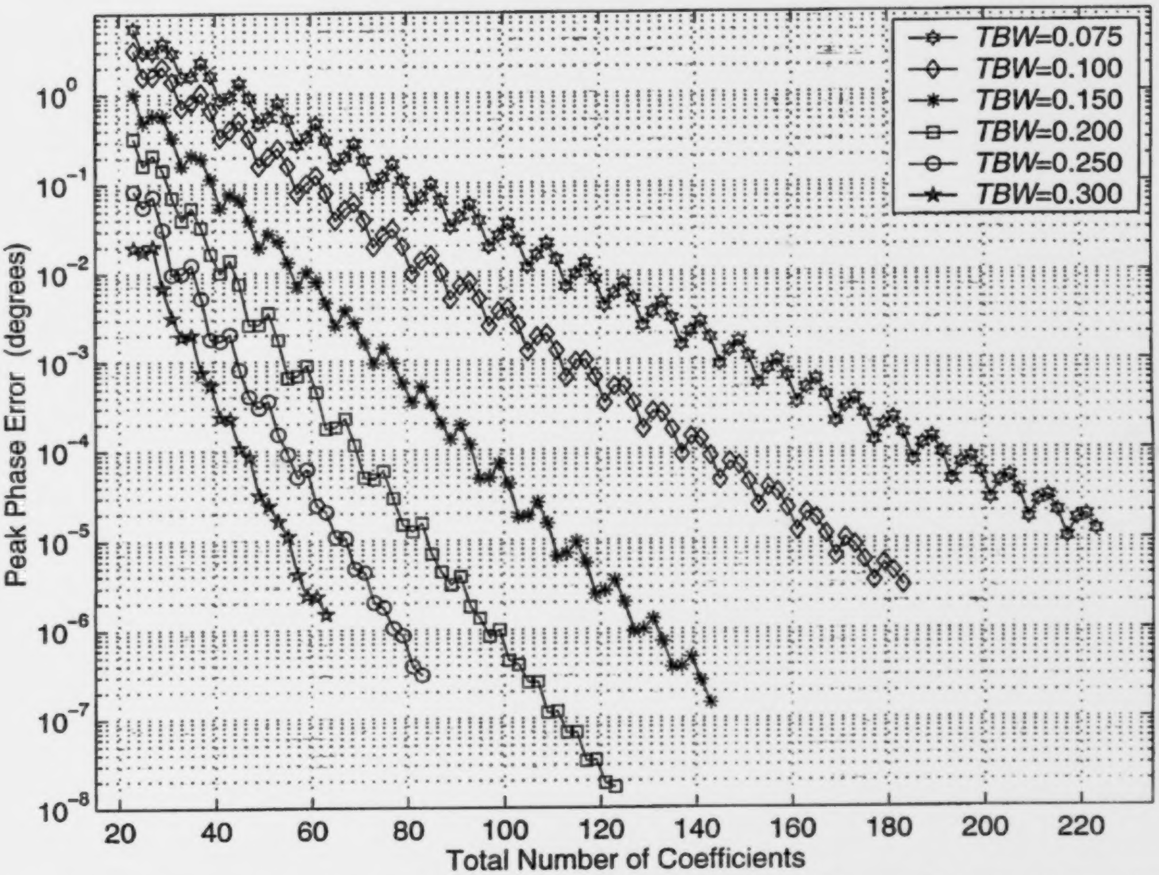

Figure 7. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of in-phase and quadrature filters. Each pair of I and Q filters was derived by sub-sampling the coefficients of a quarter-band prototype filter designed using the Parks-McClellan algorithm and the Mintzer technique. The transition band widths used in the design of the prototype filters, *TBW*, are normalized to the Nyquist frequency.

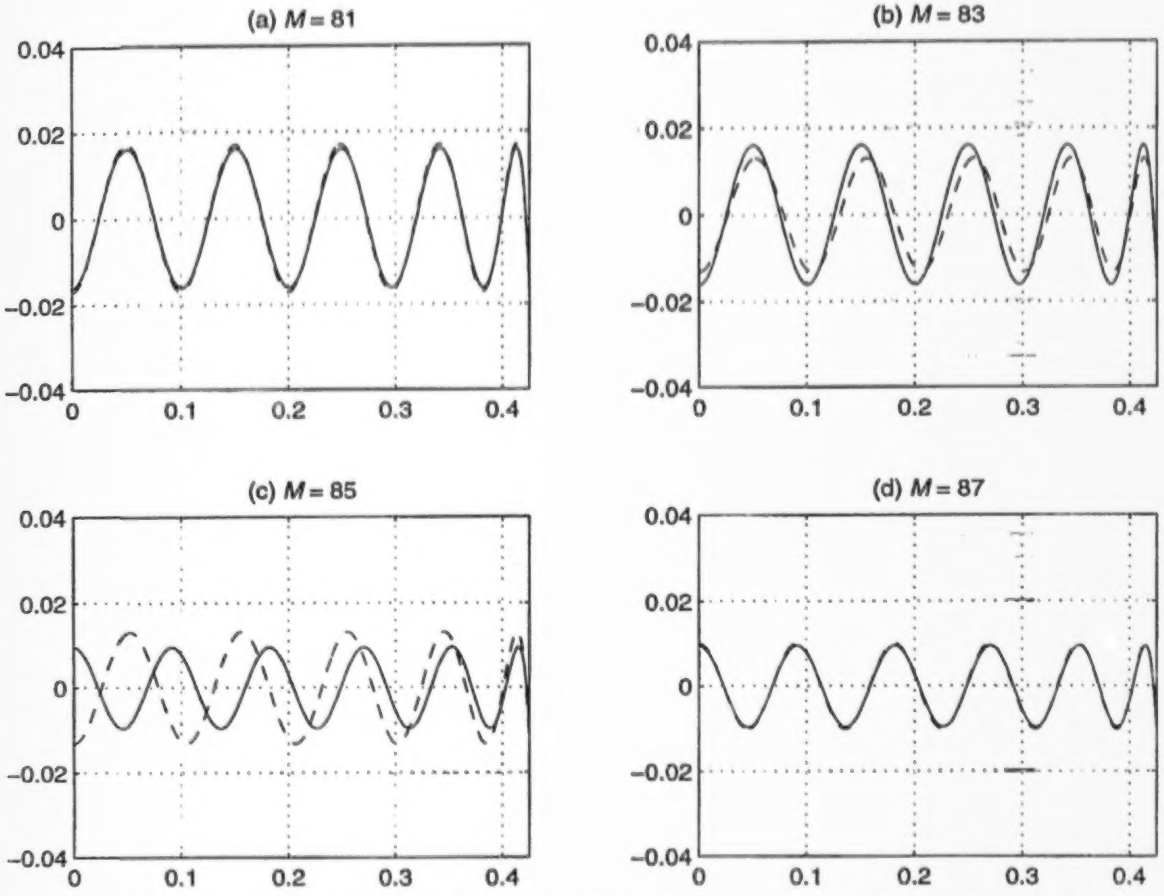

Figure 8. Example magnitude responses obtained for pairs of I and Q filters. The Parks-McClellan algorithm was used to separately design the I and Q filters to a common magnitude response specification. The transition band width, *TBW*=0.075, is normalized to the Nyquist frequency and corresponds to the transition band width of a prototype filter from which I and Q filters having the same magnitude response specifications could be derived.

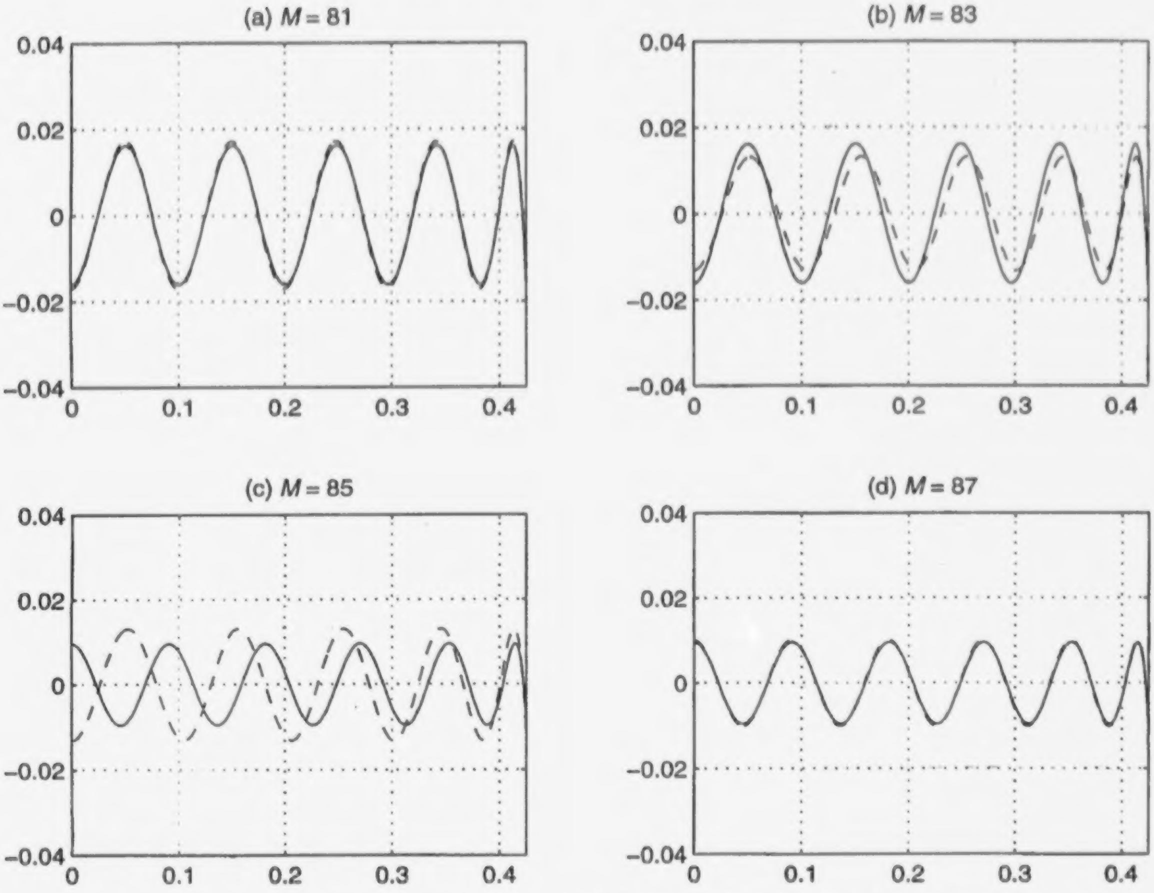

Figure 9. Example magnitude responses obtained for pairs of I and Q filters. The Parks-McClellan algorithm was used to separately design the I and Q filters with the Vaidyanathan technique being used to design the I filter to a half-band magnitude response specification. The transition band width, *TBW*=0.075, is normalized to the Nyquist frequency and corresponds to the transition band width of a prototype filter from which I and Q filters having the same magnitude response specifications could be derived.

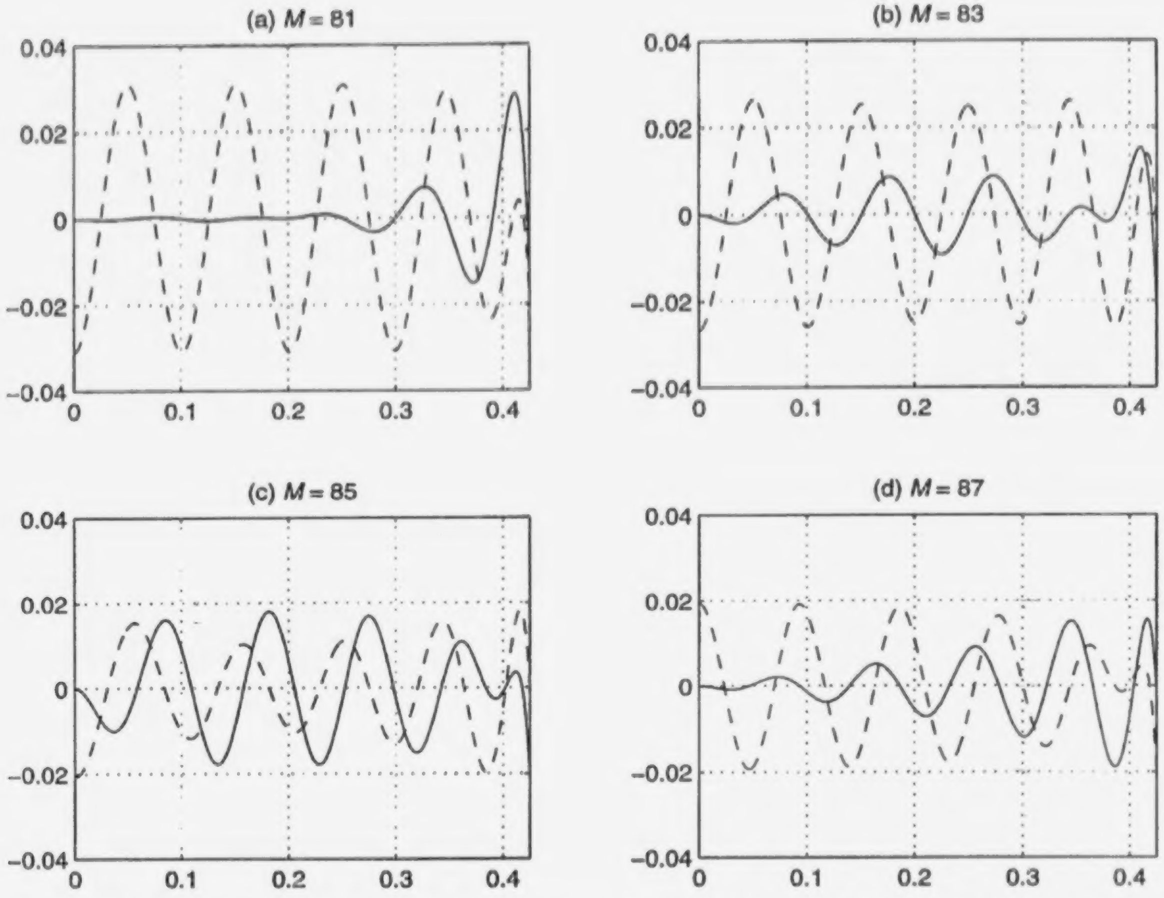

Figure 10. Example magnitude responses obtained for pairs of I and Q filters. Each pair of I and Q filters was formed by sub-sampling the filter coefficients of a prototype filter, which was designed using the Parks-McClellan algorithm. The transition band width of the prototype filter, *TBW*, was 0.075 normalized to the Nyquist frequency.

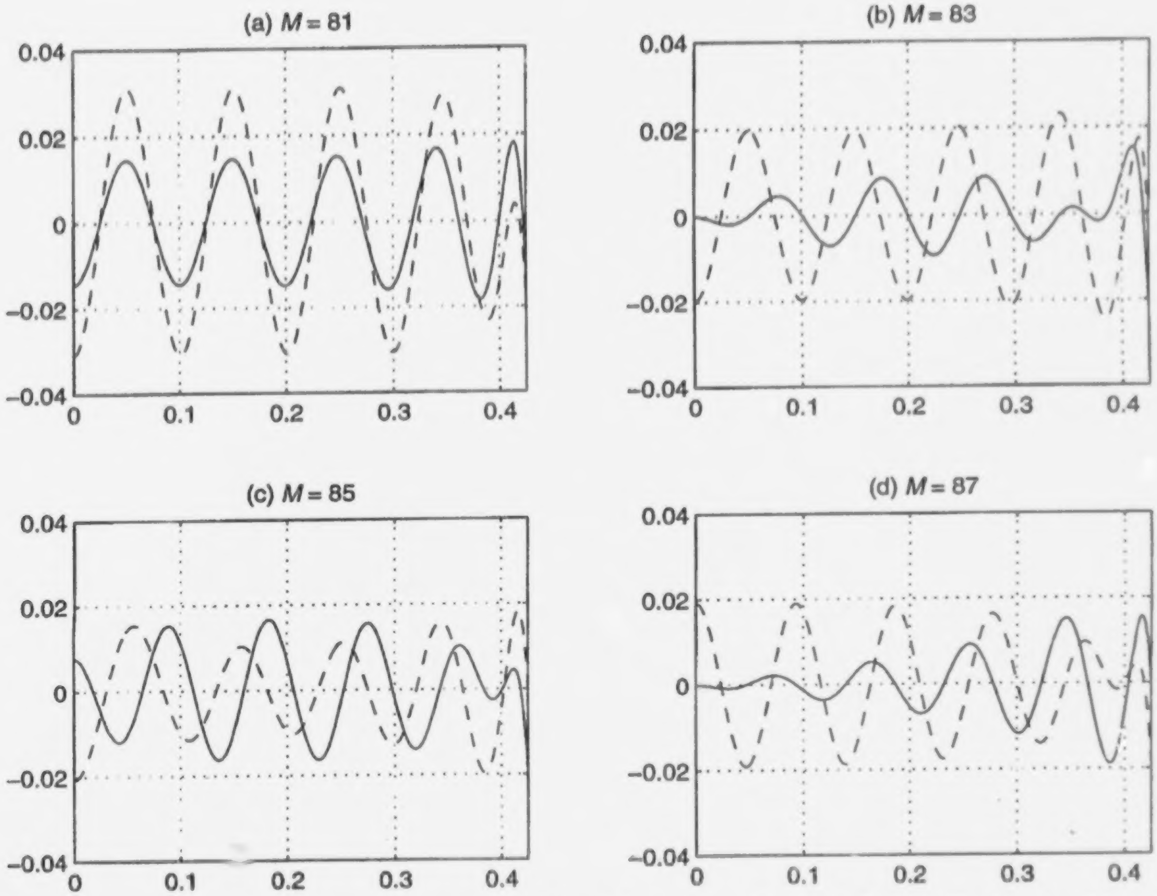

Figure 11. Example magnitude responses obtained for pairs of I and Q filters. Each pair of I and Q filters was formed by sub-sampling the filter coefficients of a filter, which was designed using the Parks-McClellan algorithm and the Mintzer technique. The transition band width of the prototype filter, *TBW*, was 0.075 normalized to the Nyquist frequency.

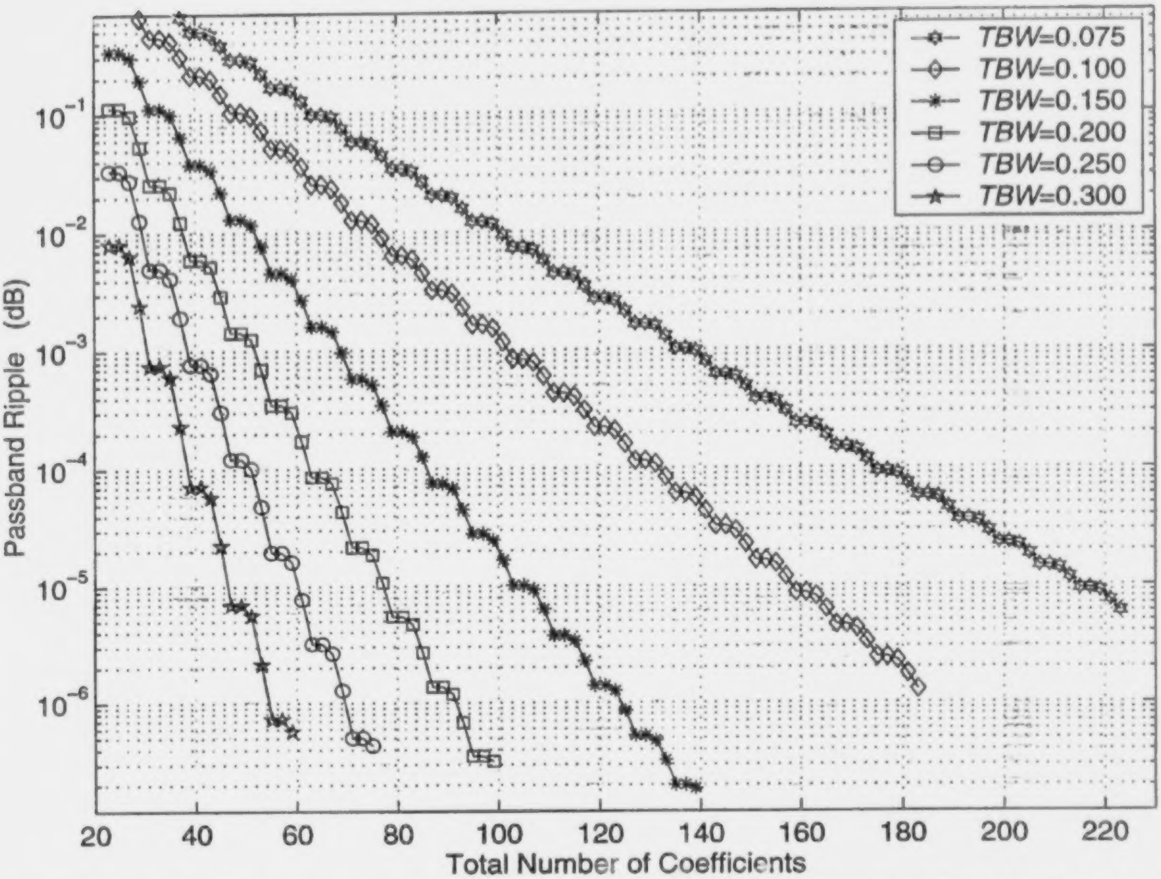

Figure 12. Peak-to-peak passband magnitude response errors, $2\delta_{IQ}$, plotted as a function of the total number of filter coefficients, *M*, for pairs of in-phase and quadrature filters. The filters were separately designed using the Parks-McClellan algorithm. Each of the transition band widths, *TBW*, is normalized to the Nyquist frequency and corresponds to the transition band width of a prototype filter from which I and Q filters having the same magnitude response specifications could be derived.

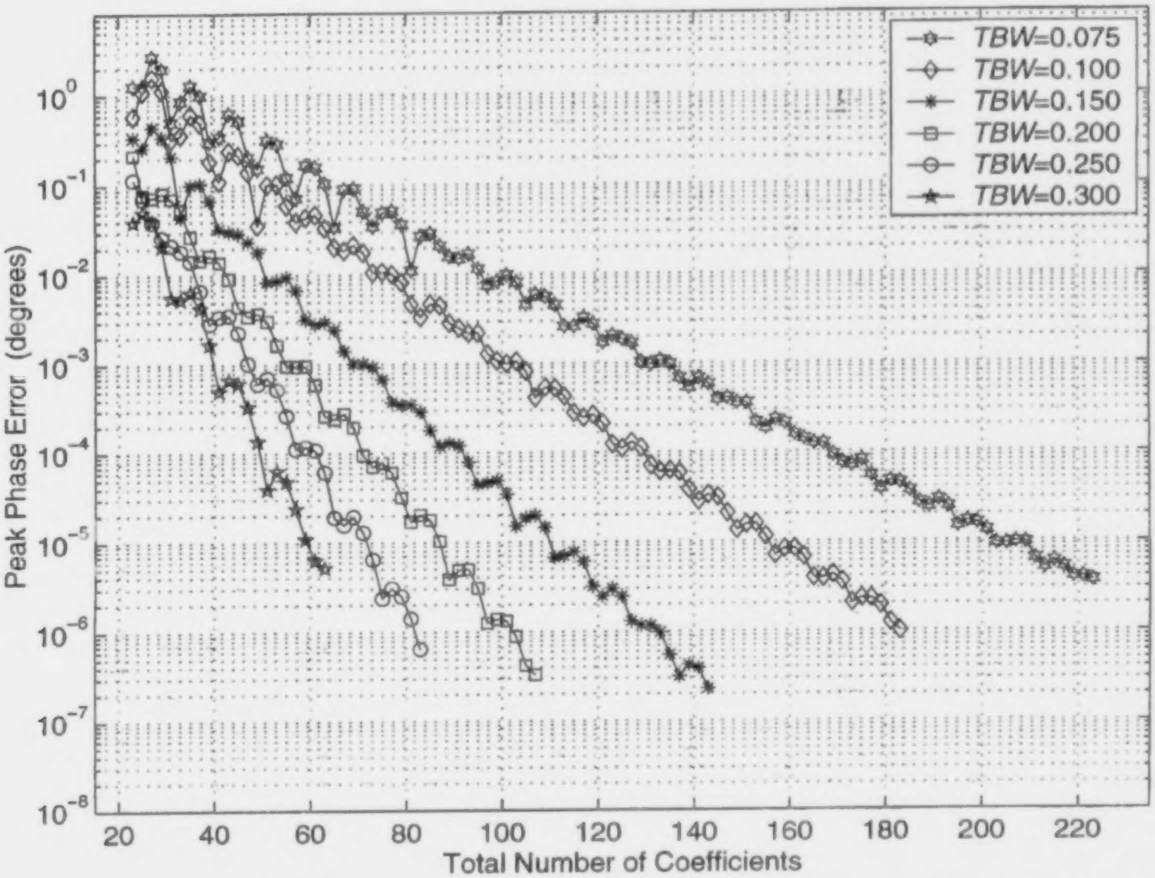

Figure 13. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of in-phase and quadrature filters. The filters were separately designed using the window method with the Kaiser window function. Each of the transition band widths, *TBW*, is normalized to the Nyquist frequency and corresponds to the transition band width of a prototype filter from which I and Q filters having the same magnitude response specifications could be derived.

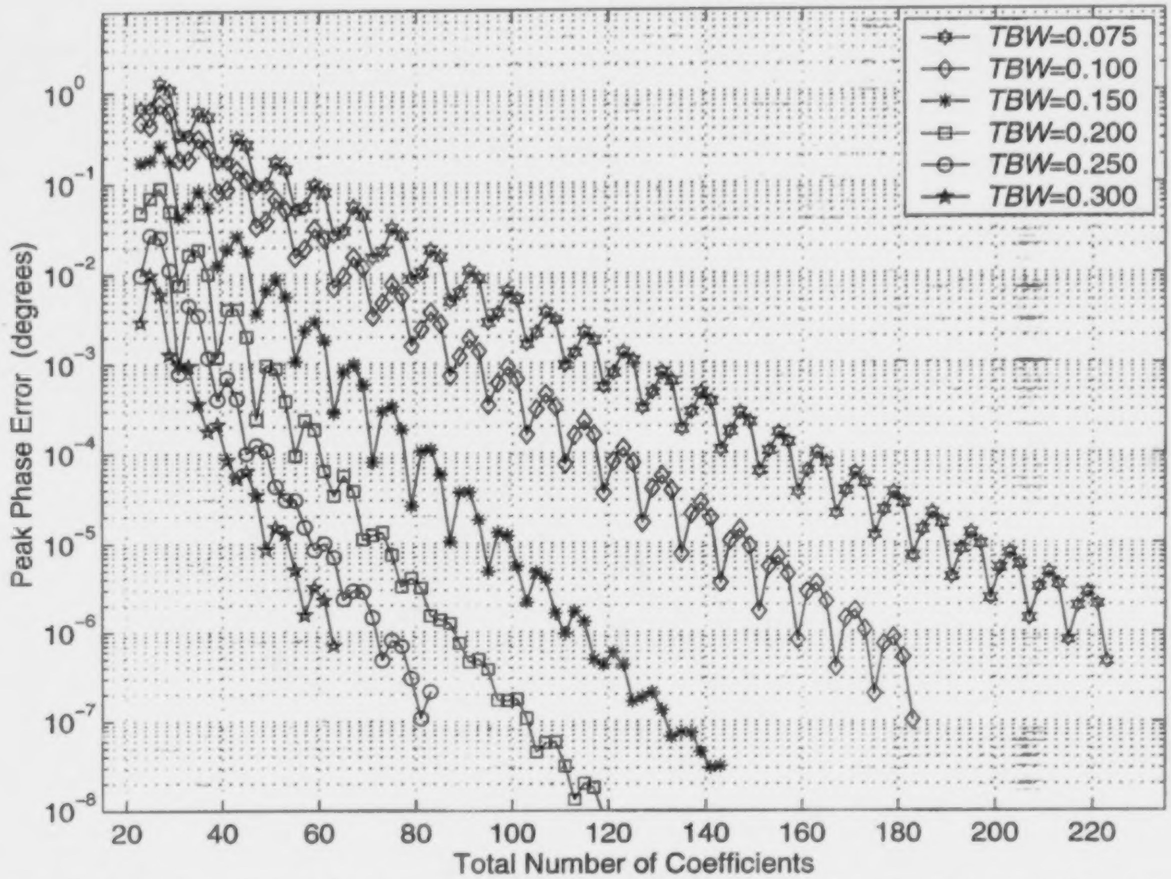

Figure 14. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of I and Q filters. Each pair of I and Q filters was formed by sub-sampling the coefficients of a prototype filter, which was designed using the window method with the Kaiser window function. The transition band widths, *TBW*, which were used to design the prototype filters, are normalized to the Nyquist frequency.

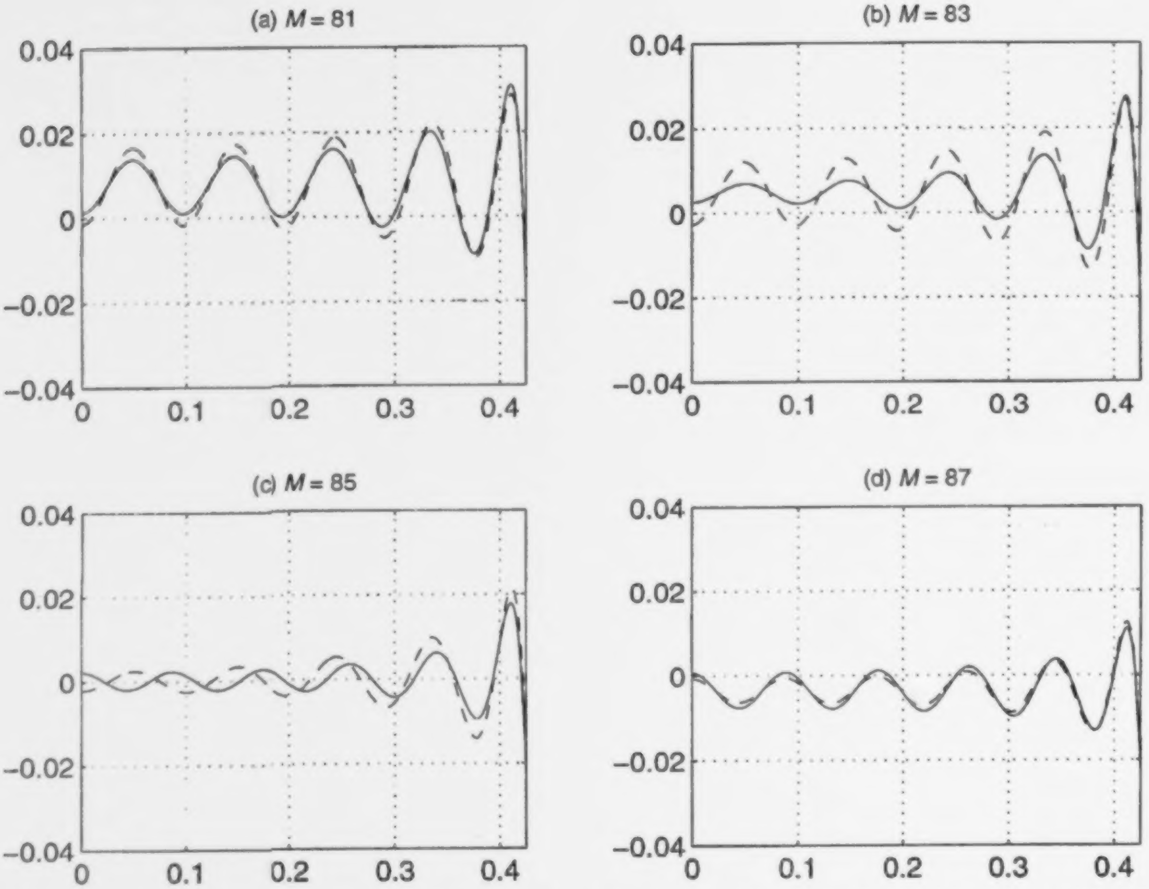

Figure 15. Example magnitude responses obtained for pairs of I and Q filters. Each pair of I and Q filters was formed by sub-sampling the coefficients of a prototype filter, which was designed using the window method with the Kaiser window function. A transition band width of *TBW*=0.075 was used to design the prototype filters.

5.4.2 Passband Magnitude Response Mismatch

The best performance results for the passband peak-to-peak magnitude response mismatch, Δ_{IQ} or alternatively, the peak phase error bound are usually obtained when the I and Q filters are separately designed using the Parks-McClellan algorithm, with or without Vaidyanathan's technique for (M-7) or (M-1) divisible by 8. However, the results for the case where (M-3) is divisible by 8 are among the worst obtained with any of the design methods considered. The performance of the I and Q filters designed using the Kaiser window was usually good for (M-7) divisible by 8, particularly for small transition bandwidths and large values of M.

5.4.3 Transition Band Magnitude Response Error

The matching of the I and Q filter magnitude responses is known to hold up best in the transition band for I and Q filters derived from a prototype filter, particularly if the Kaiser window design method is used [7].[7] If the I and Q filters are separately designed, the magnitude response mismatch degrades rapidly within the transition band, particularly if the Parks-McClellan algorithm is used to design the I and Q filters.[8] The use of the Parks-McClellan algorithm to separately design the I and Q filters gave the best control over the widths of the transition bands.

5.4.4 Stopband Magnitude Response Error

The best performance results for the stopband magnitude response ripple are obtained when the I and Q filters are separately designed using the Parks-McClellan algorithm. As is the case for the passband magnitude response ripple, this is to be expected since the Parks-McClellan algorithm is known to be an optimal design method in the minimax sense.

5.5 Effects of Other Filter Design Parameters

The results previously considered involved constraints on the choices of passband width and the relative weights applied to the pass and stopband errors when the Parks-McClellan algorithm is used. In general, there may be application requirements which favour different choices of these design parameters. Consequently, some additional results were obtained to provide some insight into the effects of varying them.

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[7] The magnitude response mismatch can be significant in applications where the quadrature demodulator is intended to limit the bandwidth of an input signal with the minimum generation of spurious signals.

[8] If the Parks-McClellan algorithm is used to separately design the I and Q filters, the algorithm optimizes the pass and stopband magnitude responses while allowing the transition band magnitude response to vary. Since the I and Q filters are designed independently, significant variations in their magnitude responses can be expected in the transition band. Alternatively, if a window method is used to separately design a pair of I and Q filters, the window functions used are slightly different.

5.5.1 I and Q Filters Separately Designed Using the Parks-McClellan Algorithm

Passband width. The dependence of the peak phase error bounds on M is shown in Figure 16 for the increased passband width of 0.3 (referenced to an equivalent prototype filter). A comparison with the corresponding results for the passband width of 0.25, shown in Figure 4, reveals that the performance dependence on M has changed. While the general trend is similar, the modulo 8 periodicity previously observed no longer holds. Note that the I filter is no longer a half-band filter and that the savings in computational cost from having zero-valued filter coefficients are lost.

Weights on pass and stopband errors. The dependence of the peak passband phase error on M for relative weights ranging from 4:1 to 1:4 for pass and stopband errors is shown in Figure 17. The choice of weights has significant overall effects on the phase error bounds and also affects the values of M at which locally optimal results are obtained. The results are consistent with the expectation that an increase in the relative weight on the passband error will reduce the passband magnitude response ripple and thereby tend to reduce the magnitude response mismatch.

These results indicate that the magnitude response matching of I and Q filters, which are separately designed using the Parks-McClellan algorithm, has a complex dependence on the design parameters including the pass and stopband widths, and weights. If the phase error, or other performance parameters dependent on the matching of the passband I and Q filter magnitude responses, are important, significant benefits may be possible by investigating the effects of small variations in the design parameters before selecting a final design.

5.5.2 I and Q Filters Derived from a Prototype Filter Designed Using the Parks-McClellan Algorithm

Weights on pass and stopband errors. The dependence of the phase error bounds on M for different relative weights applied to the pass and stopband magnitude response errors is shown in Figure 18. The effect is the reverse of that observed for the separately designed filters; an increase in the relative weight on the passband error degrades the phase error performance. Figure 19 plots example magnitude responses for a 4:1 weighing of the pass and stopband errors. The I and Q filters magnitude responses are nearly equiripple with the magnitude response mismatch approaching twice the peak-to-peak ripple, $2\delta_I$, or $2\delta_Q$, of either the I and Q filters. This result is contrary to the intuitive expectation that modifying the error weights to reduce the passband magnitude ripple should also improve the magnitude response matching.

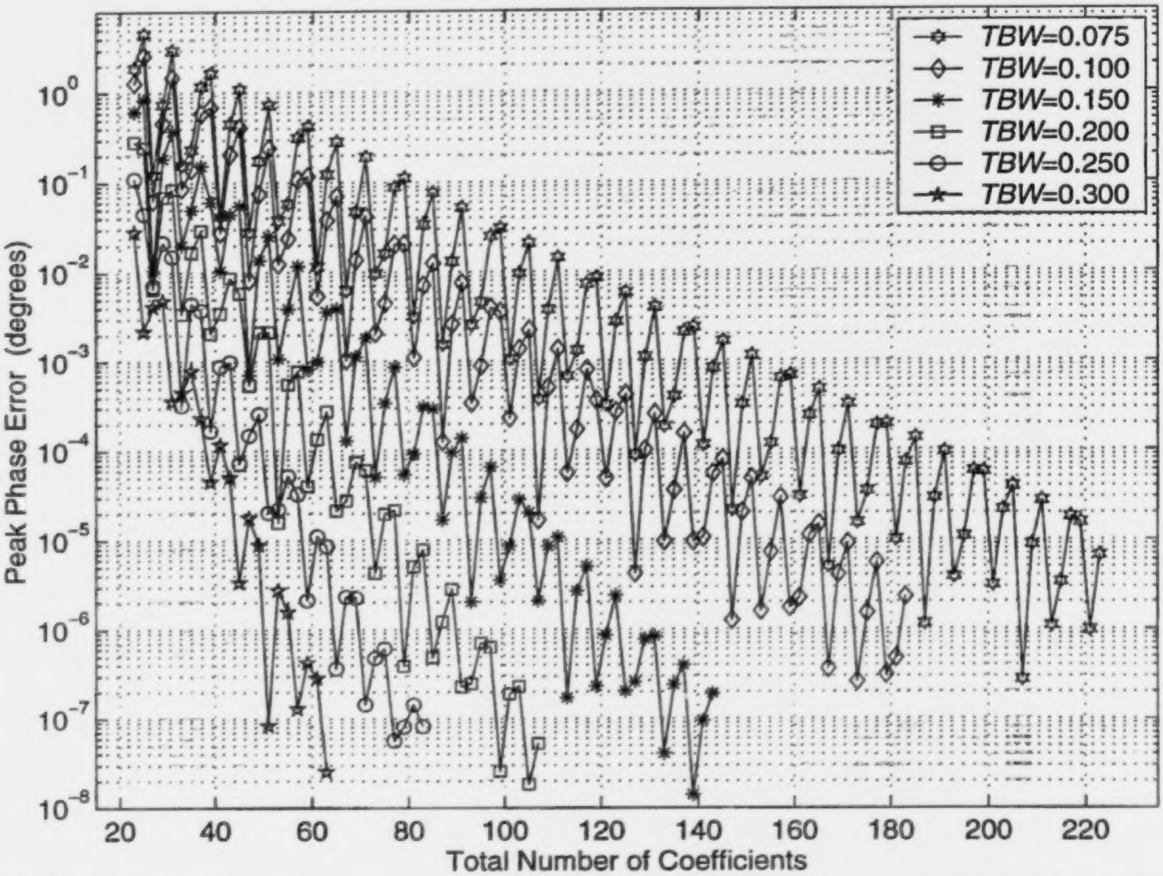

Figure 16. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of I and Q filters, which were separately designed using the Parks-McClellan algorithm. The passband width corresponds to that of I and Q filters derived from a prototype filter having a passband width of 0.3 normalized to the Nyquist frequency.

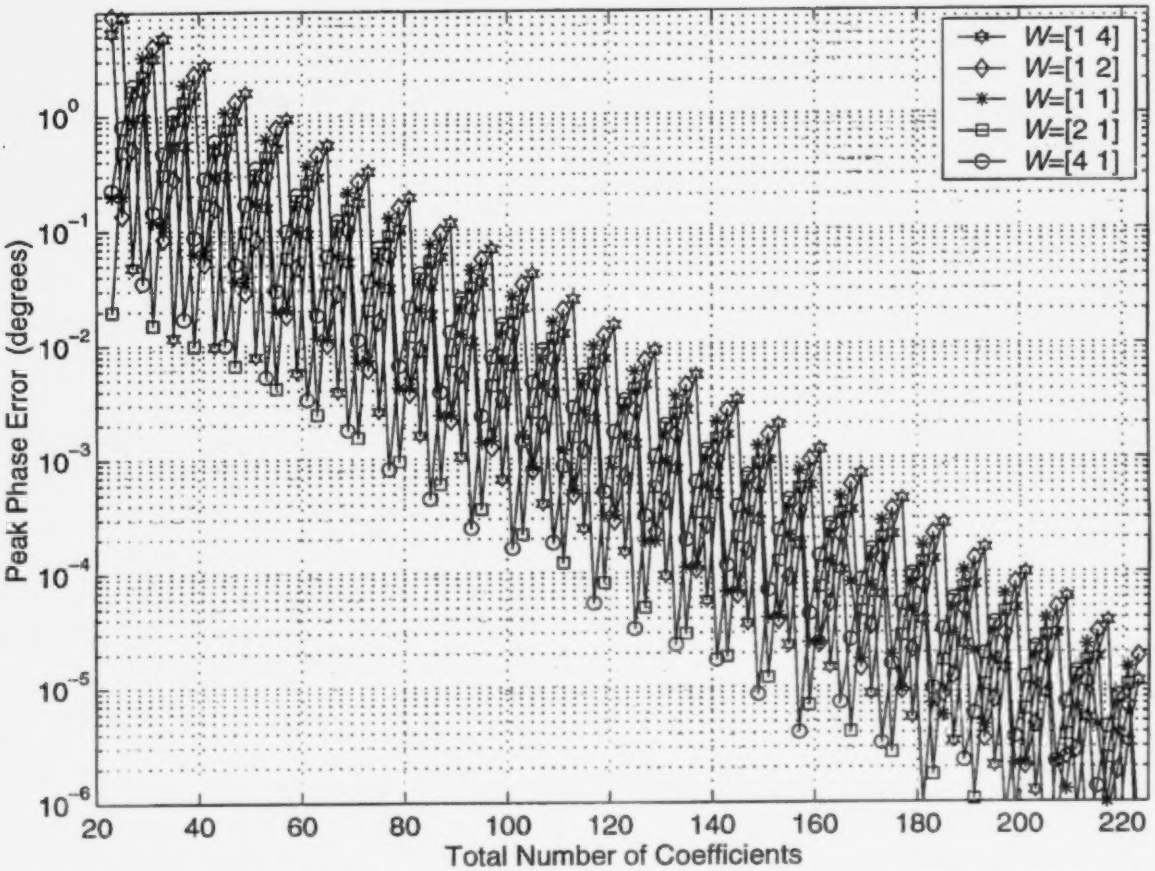

Figure 17. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of in-phase and quadrature filters. The filters were separately designed using the Parks-McClellan algorithm. The weights applied to the pass and stopband errors ranged from 1:4 to 4:1, respectively The transition band width, *TBW*=0.075, corresponds to the transition band width of a prototype filter from which I and Q filters having the same magnitude response specifications could be derived and is normalized to the Nyquist frequency.

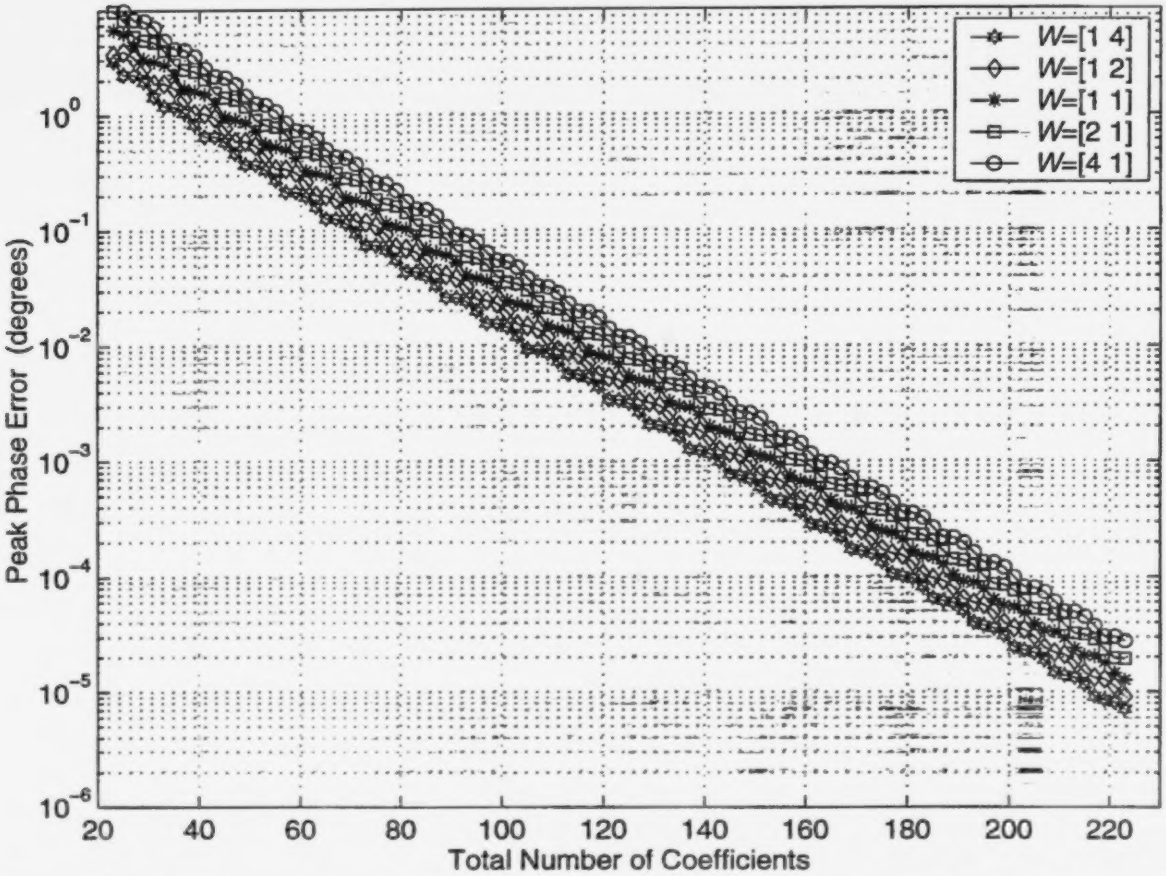

Figure 18. Dependence of phase error bounds on the total number of filter coefficients, *M*, for pairs of I and Q filters. Each pair of I and Q filters was formed by sub-sampling the filter coefficients of a prototype filter, which was designed using the Parks-McClellan algorithm. The weights applied to the pass and stopband errors ranged from 1:4 to 4:1. The transition band width, *TBW*, was 0.075 normalized to the Nyquist frequency.

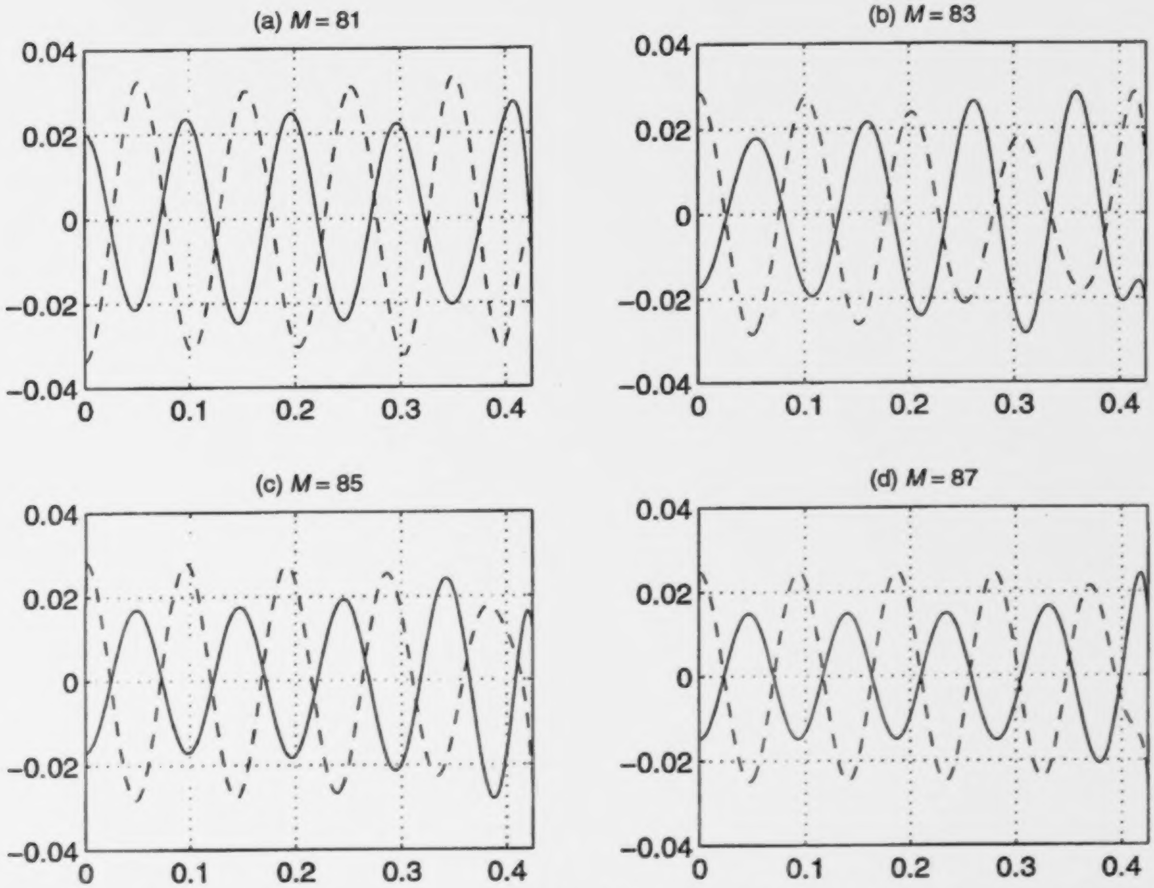

Figure 19. Example magnitude responses obtained for pairs of I and Q filters. Each pair of I and Q filters was formed by sub-sampling the filter coefficients of a prototype filter, which was designed using the Parks-McClellan algorithm. Each pair of I and Q filters was formed by sub-sampling the filter coefficients of a prototype filter, which was designed using the Parks-McClellan algorithm with relative weights of 4:1 for the pass and stopband errors. The transition band width, *TBW*, was 0.075 normalized to the Nyquist frequency.

6. CONCLUSIONS

The computationally efficient digital quadrature demodulator design considered in this paper has attractive features. A favourable tradeoff between accuracy and computational cost can be achieved if some care is taken in the design of the filters to ensure good matching of the magnitude responses. However, significant differences in the mismatch between the I and Q filter magnitude responses can occur depending on the choice of filter design method, frequency response specification and the number of filter coefficients. Consequently, some care should be taken in the selection and application of the design approach.

The McClellan-Parks algorithm is widely used designing individual FIR filters because a given magnitude response specification can be satisfied with the minimum possible peak error for a given number of filter coefficients. However, there are several issues when it is used to design I and Q filters for quadrature demodulators based on quadrature mixing and lowpass filtering approaches where a 4:1 ratio between the sampling rate and intermediate frequency is used.. The first concerns the performance dependence on the number of filter coefficients and the frequency response design specification used in the design of the I and Q filters. The second is whether the I and Q filters should be separately designed to a common magnitude response specification, or be derived from a single prototype filter. Finally, if it is desired to make the I filter a half-band filter to reduce computational cost, the desired filter design cannot be obtained directly and one of the methods for using the Parks-McClellan algorithm to construct fractional-band filters must be used.

We have found that the Parks-McClellan algorithm can be used to separately design I and Q filters whose passband magnitude responses have low ripple and good matching, but these favourable results are dependent on the choice of the total number of filter coefficients and frequency response design specification. Particularly good results can usually be achieved when the filters are designed using a half-band frequency response specification if the total number of filter coefficients, M, is chosen such that $(M-1)$ or $(M-7)$ are divisible by 8. Very similar results can be achieved when the in-phase filter is designed using Vaidyanathan's technique to obtain a true half-band filter design where nearly half the coefficients are equal to zero. This result is important since it indicates that a useful saving in computational cost can be obtained without any performance loss. However, the design approaches involving the separate design of the I and Q filters with the Parks-McClellan algorithm were all observed to suffer from a significant degradation in the matching of their magnitude responses at frequencies outside the passband.

The Parks-McClellan algorithm yielded unexceptional results when it was used to design a prototype filter from which the I and Q filters were derived. Although the performance outside the passband held up better than when the filters were separately designed, this design approach cannot be recommended.

Design approaches based on the use of the window design method are serious alternatives to approaches based on the Parks-McClellan algorithm. A good passband performance can be obtained when the I and Q filters are separately derived from a prototype filter designed using the Kaiser window function, particularly if good choices are made for the number of filter coefficients. A clear performance advantage could only be obtained with the Parks-McClellan algorithm when it was used to separately design the I and Q filters. In several respects, window design methods have advantages which can be important. The I and Q filters derived from a prototype filter designed using the Kaiser window function provided the best phase error bound performance when this was measured over the extended bandwidth $f_s'/2$ (i.e., the -6 dB bandwidth for a half-band magnitude response specification). Also, window design methods are free from the convergence problems which limit the maximum number of filter coefficients in filters designed using the Parks-McClellan algorithm.[9]

This paper was primarily concerned with I and Q filters designed to a half-band filter magnitude response specification with limited choices for the other design parameters used in the design specification. The results obtained with other design specifications differ, particularly with respect to their dependence on the number of filter coefficients. Consequently, when the Parks-McClellan algorithm is used to design I and Q filters to other magnitude response specifications, it is desirable to investigate the effect of varying the number of filter coefficients and the pass and stopband weights with the intention of selecting a pair of I and Q filters whose magnitude responses are well matched.

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[9] With the MATLAB™ implementation of the Parks-McClellan algorithm, a globally optimal number of filter coefficients could be found; further increases in the number of filter coefficients always resulting in a significant degradation of the phase error performance. Since this behaviour was not accompanied by error messages or other obvious indications of problems, care should be taken when using the Parks-McClellan algorithm to avoid this problem.

7. REFERENCES

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APPENDIX A - PERFORMANCE DATA FOR QUADRATURE DEMODULATION FILTERS SEPARATELY DESIGNED USING THE PARKS-McCLELLAN ALGORITHM

Table A.1. Performance data for I and Q filters where $TBW = 0.075$.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.4250	0.4251	0.4250	-0.5992	-0.7600	-0.5992	0.7916
23	0.4250	0.4251	0.4250	-0.7323	-0.7600	-0.7323	-0.7006
25	0.4250	0.4250	0.4250	-0.7323	-0.7602	-0.7323	-0.6948
27	0.4251	0.4250	0.4250	-0.8625	-0.7602	-0.7602	-0.0344
29	0.4251	0.4250	0.4251	-0.8625	-1.0150	-0.8625	0.5144
31	0.4250	0.4250	0.4250	-0.9901	-1.0150	-0.9901	-0.9175
33	0.4250	0.4251	0.4250	-0.9901	-1.0154	-0.9901	-0.9634
35	0.4251	0.4251	0.4251	-1.1150	-1.0154	-1.0154	-0.2535
37	0.4251	0.4251	0.4251	-1.1150	-1.2631	-1.1150	0.2755
39	0.4251	0.4251	0.4251	-1.2378	-1.2631	-1.2378	-1.2024
41	0.4251	0.4251	0.4251	-1.2378	-1.2628	-1.2378	-1.2041
43	0.4250	0.4251	0.4251	-1.3591	-1.2628	-1.2628	-0.5235
45	0.4250	0.4250	0.4250	-1.3591	-1.5037	-1.3591	0.0293
47	0.4251	0.4250	0.4251	-1.4785	-1.5037	-1.4785	-1.4431
49	0.4251	0.4250	0.4251	-1.4785	-1.5024	-1.4785	-1.4482
51	0.4251	0.4250	0.4250	-1.5979	-1.5024	-1.5024	-0.7584
53	0.4251	0.4250	0.4251	-1.5979	-1.7395	-1.5979	-0.2085
55	0.4250	0.4250	0.4250	-1.7148	-1.7395	-1.7148	-1.7043
57	0.4250	0.4250	0.4250	-1.7148	-1.7396	-1.7148	-1.6904
59	0.4250	0.4250	0.4250	-1.8318	-1.7396	-1.7396	-0.9967
61	0.4250	0.4250	0.4250	-1.8318	-1.9726	-1.8318	-0.4415
63	0.4250	0.4250	0.4250	-1.9490	-1.9726	-1.9490	-1.9379
65	0.4250	0.4250	0.4250	-1.9490	-1.9717	-1.9490	-1.9459
67	0.4250	0.4250	0.4250	-2.0644	-1.9717	-1.9717	-1.2217
69	0.4250	0.4250	0.4250	-2.0644	-2.2012	-2.0644	-0.6733
71	0.4250	0.4250	0.4250	-2.1781	-2.2012	-2.1781	-2.1552
73	0.4250	0.4250	0.4250	-2.1781	-2.2030	-2.1781	-2.1355
75	0.4250	0.4250	0.4250	-2.2924	-2.2030	-2.2030	-1.4586
77	0.4250	0.4250	0.4250	-2.2924	-2.4285	-2.2924	-0.8992

79	0.4250	0.4250	0.4250	-2.4055	-2.4285	-2.4055	-2.3788
81	0.4250	0.4250	0.4250	-2.4055	-2.4286	-2.4055	-2.3796
83	0.4250	0.4250	0.4250	-2.5189	-2.4286	-2.4286	-1.6819
85	0.4250	0.4250	0.4250	-2.5189	-2.6554	-2.5189	-1.1256
87	0.4250	0.4250	0.4250	-2.6317	-2.6554	-2.6317	-2.6173
89	0.4250	0.4250	0.4250	-2.6317	-2.6548	-2.6317	-2.6149
91	0.4250	0.4250	0.4250	-2.7446	-2.6548	-2.6548	-1.9068
93	0.4250	0.4250	0.4250	-2.7446	-2.8791	-2.7446	-1.3510
95	0.4250	0.4250	0.4250	-2.8565	-2.8791	-2.8565	-2.8577
97	0.4250	0.4250	0.4250	-2.8565	-2.8765	-2.8565	-2.8423
99	0.4250	0.4250	0.4250	-2.9674	-2.8765	-2.8765	-2.1355
101	0.4250	0.4250	0.4250	-2.9674	-3.1004	-2.9674	-1.5822
103	0.4250	0.4250	0.4250	-3.0780	-3.1004	-3.0780	-2.7916
105	0.4250	0.4250	0.4250	-3.0780	-3.1013	-3.0780	-3.0717
107	0.4250	0.4250	0.4250	-3.1861	-3.1013	-3.1013	-2.3582
109	0.4250	0.4250	0.4250	-3.1861	-3.3194	-3.1861	-1.8039
111	0.4249	0.4250	0.4249	-3.2980	-3.3194	-3.2980	-2.9243
113	0.4249	0.4249	0.4249	-3.2980	-3.3223	-3.2980	-3.2736
115	0.4249	0.4249	0.4249	-3.4104	-3.3223	-3.3223	-2.5810
117	0.4249	0.4249	0.4249	-3.4104	-3.5436	-3.4104	-2.0163
119	0.4249	0.4249	0.4249	-3.5206	-3.5436	-3.5206	-3.4978
121	0.4249	0.4249	0.4249	-3.5206	-3.5432	-3.5206	-3.4953
123	0.4249	0.4249	0.4249	-3.6307	-3.5432	-3.5432	-2.7983
125	0.4249	0.4249	0.4249	-3.6307	-3.7650	-3.6307	-2.2366
127	0.4249	0.4249	0.4249	-3.7424	-3.7650	-3.7424	-3.7228
129	0.4249	0.4249	0.4249	-3.7424	-3.7606	-3.7424	-3.7202
131	0.4249	0.4249	0.4249	-3.8486	-3.7606	-3.7606	-3.0205
133	0.4249	0.4249	0.4249	-3.8486	-3.9819	-3.8486	-2.4574
135	0.4249	0.4249	0.4249	-3.9584	-3.9819	-3.9584	-3.9499
137	0.4249	0.4249	0.4249	-3.9584	-3.9816	-3.9584	-3.9443
139	0.4249	0.4249	0.4249	-4.0680	-3.9816	-3.9816	-3.2386
141	0.4249	0.4249	0.4249	-4.0680	-4.2033	-4.0680	-2.6774
143	0.4249	0.4249	0.4249	-4.1812	-4.2033	-4.1812	-4.1650
145	0.4249	0.4249	0.4249	-4.1812	-4.2023	-4.1812	-4.1544

147	0.4249	0.4249	0.4249	-4.2879	-4.2023	-4.2023	-3.4582
149	0.4249	0.4249	0.4249	-4.2879	-4.4183	-4.2879	-2.8934
151	0.4249	0.4249	0.4249	-4.3948	-4.4183	-4.3948	-4.3935
153	0.4249	0.4249	0.4249	-4.3948	-4.4194	-4.3948	-4.3787
155	0.4248	0.4249	0.4249	-4.5060	-4.4194	-4.4194	-3.6759
157	0.4248	0.4248	0.4248	-4.5060	-4.6343	-4.5060	-3.1104
159	0.4248	0.4248	0.4248	-4.6135	-4.6343	-4.6135	-4.6089
161	0.4248	0.4248	0.4248	-4.6135	-4.6317	-4.6135	-4.6118
163	0.4248	0.4248	0.4248	-4.7161	-4.6317	-4.6317	-3.8964
165	0.4248	0.4248	0.4248	-4.7161	-4.8451	-4.7161	-3.3352
167	0.4248	0.4248	0.4248	-4.8245	-4.8451	-4.8245	-4.0813
169	0.4248	0.4248	0.4248	-4.8245	-4.8524	-4.8245	-4.7550
171	0.4248	0.4248	0.4248	-4.9365	-4.8524	-4.8524	-4.1113
173	0.4248	0.4248	0.4248	-4.9365	-5.0685	-4.9365	-3.5450
175	0.4248	0.4248	0.4248	-5.0464	-5.0685	-5.0464	-4.7179
177	0.4248	0.4248	0.4248	-5.0464	-5.0683	-5.0464	-5.0183
179	0.4248	0.4248	0.4248	-5.1528	-5.0683	-5.0683	-4.3212
181	0.4248	0.4248	0.4248	-5.1528	-5.2840	-5.1528	-3.7622
183	0.4248	0.4248	0.4248	-5.2626	-5.2840	-5.2626	-5.1457
185	0.4248	0.4248	0.4248	-5.2626	-5.2803	-5.2626	-5.2492
187	0.4248	0.4248	0.4248	-5.3715	-5.2803	-5.2803	-4.5398
189	0.4248	0.4247	0.4248	-5.3715	-5.5009	-5.3715	-3.9822
191	0.4247	0.4247	0.4247	-5.4791	-5.5009	-5.4791	-4.6841
193	0.4247	0.4247	0.4247	-5.4791	-5.5011	-5.4791	-5.3336
195	0.4247	0.4247	0.4247	-5.5870	-5.5011	-5.5011	-4.7579
197	0.4247	0.4247	0.4247	-5.5870	-5.7151	-5.5870	-4.1898
199	0.4247	0.4247	0.4247	-5.6920	-5.7151	-5.6920	-5.6860
201	0.4247	0.4247	0.4247	-5.6920	-5.7121	-5.6920	-5.6712
203	0.4247	0.4247	0.4247	-5.8000	-5.7121	-5.7121	-4.9764
205	0.4247	0.4247	0.4247	-5.8000	-5.9298	-5.8000	-4.4056
207	0.4247	0.4247	0.4247	-5.9082	-5.9298	-5.9082	-5.6505
209	0.4247	0.4247	0.4247	-5.9082	-5.9324	-5.9082	-5.8946
211	0.4247	0.4247	0.4247	-6.0180	-5.9324	-5.9324	-5.1880
213	0.4247	0.4247	0.4247	-6.0180	-6.1478	-6.0180	-4.6310

215	0.4247	0.4247	0.4247	-6.1254	-6.1478	-6.1254	-5.3346
217	0.4247	0.4247	0.4247	-6.1254	-6.1449	-6.1254	-5.9994
219	0.4247	0.4247	0.4247	-6.2320	-6.1449	-6.1449	-5.3977
221	0.4247	0.4246	0.4247	-6.2320	-6.3593	-6.2320	-4.8348
223	0.4246	0.4246	0.4246	-6.3373	-6.3593	-6.3373	-6.3206

Table A.2. Performance summary for I and Q filters where *TBW*= 0.150.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.3500	0.3500	0.3500	-1.1612	-1.4591	-1.1612	0.1755
23	0.3500	0.3500	0.3500	-1.4088	-1.4591	-1.4088	-1.1965
25	0.3500	0.3501	0.3500	-1.4088	-1.4579	-1.4088	-1.2007
27	0.3501	0.3501	0.3501	-1.6509	-1.4579	-1.4579	-0.6632
29	0.3501	0.3500	0.3501	-1.6509	-1.9404	-1.6509	-0.3176
31	0.3500	0.3500	0.3500	-1.8918	-1.9404	-1.8918	-1.6827
33	0.3500	0.3501	0.3500	-1.8918	-1.9398	-1.8918	-1.6930
35	0.3500	0.3501	0.3501	-2.1288	-1.9398	-1.9398	-1.1293
37	0.3500	0.3500	0.3500	-2.1288	-2.4096	-2.1288	-0.7900
39	0.3500	0.3500	0.3500	-2.3626	-2.4096	-2.3626	-2.1715
41	0.3500	0.3501	0.3500	-2.3626	-2.4088	-2.3626	-2.1682
43	0.3500	0.3501	0.3501	-2.5943	-2.4088	-2.4088	-1.6123
45	0.3500	0.3500	0.3500	-2.5943	-2.8719	-2.5943	-1.2561
47	0.3500	0.3500	0.3500	-2.8255	-2.8719	-2.8255	-2.6347
49	0.3500	0.3500	0.3500	-2.8255	-2.8710	-2.8255	-2.6353
51	0.3501	0.3500	0.3500	-3.0497	-2.8710	-2.8710	-2.0686
53	0.3501	0.3500	0.3501	-3.0497	-3.3234	-3.0497	-1.7325
55	0.3500	0.3500	0.3500	-3.2801	-3.3234	-3.2801	-2.8276
57	0.3500	0.3500	0.3500	-3.2801	-3.3286	-3.2801	-3.0628
59	0.3500	0.3500	0.3500	-3.5102	-3.3286	-3.3286	-2.5264
61	0.3500	0.3500	0.3500	-3.5102	-3.7811	-3.5102	-2.1689
63	0.3500	0.3500	0.3500	-3.7346	-3.7811	-3.7346	-3.5455
65	0.3500	0.3500	0.3500	-3.7346	-3.7783	-3.7346	-3.5456
67	0.3500	0.3500	0.3500	-3.9574	-3.7783	-3.7783	-2.9772
69	0.3500	0.3500	0.3500	-3.9574	-4.2266	-3.9574	-2.6226
71	0.3500	0.3500	0.3500	-4.1821	-4.2266	-4.1821	-4.0099
73	0.3500	0.3500	0.3500	-4.1821	-4.2314	-4.1821	-3.9456
75	0.3500	0.3500	0.3500	-4.4088	-4.2314	-4.2314	-3.4322
77	0.3500	0.3500	0.3500	-4.4088	-4.6768	-4.4088	-3.0690
79	0.3500	0.3500	0.3500	-4.6319	-4.6768	-4.6319	-4.4458
81	0.3500	0.3500	0.3500	-4.6319	-4.6774	-4.6319	-4.4284
83	0.3500	0.3500	0.3500	-4.8485	-4.6774	-4.6774	-3.8844

85	0.3500	0.3500	0.3500	-4.8485	-5.1196	-4.8485	-3.5243
87	0.3500	0.3500	0.3500	-5.0767	-5.1196	-5.0767	-4.6656
89	0.3500	0.3500	0.3500	-5.0767	-5.1206	-5.0767	-4.8811
91	0.3500	0.3500	0.3500	-5.3014	-5.1206	-5.1206	-4.3220
93	0.3500	0.3500	0.3500	-5.3014	-5.5541	-5.3014	-3.9708
95	0.3500	0.3500	0.3500	-5.5128	-5.5541	-5.5128	-4.8323
97	0.3500	0.3500	0.3500	-5.5128	-5.5654	-5.5128	-5.1233
99	0.3500	0.3500	0.3500	-5.7434	-5.5654	-5.5654	-4.7690
101	0.3500	0.3500	0.3500	-5.7434	-6.0087	-5.7434	-4.4010
103	0.3500	0.3500	0.3500	-5.9635	-6.0087	-5.9635	-5.7659
105	0.3500	0.3500	0.3500	-5.9635	-5.9997	-5.9635	-5.7659
107	0.3500	0.3500	0.3500	-6.1696	-5.9997	-5.9997	-5.2107
109	0.3500	0.3500	0.3500	-6.1696	-6.4467	-6.1696	-4.8459
111	0.3499	0.3500	0.3499	-6.4032	-6.4467	-6.4032	-6.2312
113	0.3499	0.3499	0.3499	-6.4032	-6.4525	-6.4032	-6.1622
115	0.3499	0.3499	0.3499	-6.6278	-6.4525	-6.4525	-5.6518
117	0.3499	0.3499	0.3499	-6.6278	-6.8947	-6.6278	-5.2863
119	0.3499	0.3499	0.3499	-6.8503	-6.8947	-6.8503	-6.6707
121	0.3499	0.3499	0.3499	-6.8503	-6.8860	-6.8503	-6.4079
123	0.3499	0.3499	0.3499	-7.0678	-6.8860	-6.8860	-6.0883
125	0.3499	0.3499	0.3499	-7.0678	-7.3349	-7.0678	-5.7264
127	0.3499	0.3499	0.3499	-7.2911	-7.3349	-7.2911	-7.1131
129	0.3499	0.3499	0.3499	-7.2911	-7.3326	-7.2911	-7.0176
131	0.3499	0.3499	0.3499	-7.5010	-7.3326	-7.3326	-6.5340
133	0.3499	0.3499	0.3499	-7.5010	-7.7688	-7.5010	-6.1648
135	0.3499	0.3499	0.3499	-7.7254	-7.7688	-7.7254	-7.5433

APPENDIX B - PERFORMANCE DATA FOR QUADRATURE DEMODULATION FILTERS SEPARATELY DESIGNED USING THE PARKS-McCLELLAN ALGORITHM (VAIDYANATHAN TECHNIQUE)

Table B.1. Performance summary for I and Q filters where *TBW*= 0.075.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.4250	0.4251	0.4250	-0.5992	-0.7600	-0.5992	0.7916
23	0.4250	0.4251	0.4250	-0.7323	-0.7600	-0.7323	-0.7002
25	0.4250	0.4251	0.4250	-0.7323	-0.7600	-0.7323	-0.7002
27	0.4251	0.4251	0.4251	-0.8625	-0.7600	-0.7600	-0.0333
29	0.4251	0.4250	0.4251	-0.8625	-1.0157	-0.8625	0.5182
31	0.4250	0.4250	0.4250	-0.9901	-1.0157	-0.9901	-0.9506
33	0.4250	0.4250	0.4250	-0.9901	-1.0157	-0.9901	-0.9506
35	0.4251	0.4250	0.4250	-1.1150	-1.0157	-1.0157	-0.2557
37	0.4251	0.4251	0.4251	-1.1150	-1.2631	-1.1150	0.2755
39	0.4251	0.4251	0.4251	-1.2378	-1.2631	-1.2378	-1.2024
41	0.4251	0.4251	0.4251	-1.2378	-1.2631	-1.2378	-1.2024
43	0.4250	0.4251	0.4251	-1.3591	-1.2631	-1.2631	-0.5237
45	0.4250	0.4250	0.4250	-1.3591	-1.5033	-1.3591	0.0295
47	0.4251	0.4250	0.4251	-1.4785	-1.5033	-1.4785	-1.4485
49	0.4251	0.4250	0.4251	-1.4785	-1.5033	-1.4785	-1.4485
51	0.4251	0.4250	0.4250	-1.5979	-1.5033	-1.5033	-0.7580
53	0.4251	0.4250	0.4251	-1.5979	-1.7398	-1.5979	-0.2063
55	0.4250	0.4250	0.4250	-1.7148	-1.7398	-1.7148	-1.6991
57	0.4250	0.4250	0.4250	-1.7148	-1.7398	-1.7148	-1.6991
59	0.4250	0.4250	0.4250	-1.8318	-1.7398	-1.7398	-0.9958
61	0.4250	0.4250	0.4250	-1.8318	-1.9721	-1.8318	-0.4413
63	0.4250	0.4250	0.4250	-1.9490	-1.9721	-1.9490	-1.9445
65	0.4250	0.4250	0.4250	-1.9490	-1.9721	-1.9490	-1.9445
67	0.4250	0.4250	0.4250	-2.0644	-1.9721	-1.9721	-1.2224
69	0.4250	0.4250	0.4250	-2.0644	-2.2012	-2.0644	-0.6713
71	0.4250	0.4250	0.4250	-2.1781	-2.2012	-2.1781	-2.1520
73	0.4250	0.4250	0.4250	-2.1781	-2.2012	-2.1781	-2.1520
75	0.4250	0.4250	0.4250	-2.2924	-2.2012	-2.2012	-1.4578
77	0.4250	0.4250	0.4250	-2.2924	-2.4285	-2.2924	-0.8992

79	0.4250	0.4250	0.4250	-2.4055	-2.4285	-2.4055	-2.3788
81	0.4250	0.4250	0.4250	-2.4055	-2.4285	-2.4055	-2.3788
83	0.4250	0.4250	0.4250	-2.5189	-2.4285	-2.4285	-1.6820
85	0.4250	0.4250	0.4250	-2.5189	-2.6547	-2.5189	-1.1253
87	0.4250	0.4250	0.4250	-2.6317	-2.6547	-2.6317	-2.6259
89	0.4250	0.4250	0.4250	-2.6317	-2.6547	-2.6317	-2.6259
91	0.4250	0.4250	0.4250	-2.7446	-2.6547	-2.6547	-1.9059
93	0.4250	0.4250	0.4250	-2.7446	-2.8785	-2.7446	-1.3507
95	0.4250	0.4250	0.4250	-2.8565	-2.8785	-2.8565	-2.8656
97	0.4250	0.4250	0.4250	-2.8565	-2.8785	-2.8565	-2.8656
99	0.4250	0.4250	0.4250	-2.9674	-2.8785	-2.8785	-2.1321
101	0.4250	0.4250	0.4250	-2.9674	-3.1013	-2.9674	-1.5743
103	0.4250	0.4250	0.4250	-3.0780	-3.1013	-3.0780	-3.0600
105	0.4250	0.4250	0.4250	-3.0780	-3.1013	-3.0780	-3.0600
107	0.4250	0.4250	0.4250	-3.1861	-3.1013	-3.1013	-2.3595
109	0.4250	0.4250	0.4250	-3.1861	-3.3207	-3.1861	-1.7965
111	0.4249	0.4250	0.4249	-3.2980	-3.3207	-3.2980	-3.2692
113	0.4249	0.4250	0.4249	-3.2980	-3.3207	-3.2980	-3.2692
115	0.4249	0.4250	0.4250	-3.4104	-3.3207	-3.3207	-2.5829
117	0.4249	0.4249	0.4249	-3.4104	-3.5436	-3.4104	-2.0163
119	0.4249	0.4249	0.4249	-3.5206	-3.5436	-3.5206	-3.4978
121	0.4249	0.4249	0.4249	-3.5206	-3.5436	-3.5206	-3.4978
123	0.4249	0.4249	0.4249	-3.6307	-3.5436	-3.5436	-2.8005
125	0.4249	0.4249	0.4249	-3.6307	-3.7647	-3.6307	-2.2365
127	0.4249	0.4249	0.4249	-3.7424	-3.7647	-3.7424	-3.7262
129	0.4249	0.4249	0.4249	-3.7424	-3.7647	-3.7424	-3.7262
131	0.4249	0.4249	0.4249	-3.8486	-3.7647	-3.7647	-3.0182
133	0.4249	0.4249	0.4249	-3.8486	-3.9817	-3.8486	-2.4573
135	0.4249	0.4249	0.4249	-3.9584	-3.9817	-3.9584	-3.9516
137	0.4249	0.4249	0.4249	-3.9584	-3.9817	-3.9584	-3.9516
139	0.4249	0.4249	0.4249	-4.0680	-3.9817	-3.9817	-3.2396
141	0.4249	0.4249	0.4249	-4.0680	-4.2030	-4.0680	-2.6773
143	0.4249	0.4249	0.4249	-4.1812	-4.2030	-4.1812	-4.1688
145	0.4249	0.4249	0.4249	-4.1812	-4.2030	-4.1812	-4.1688

147	0.4249	0.4249	0.4249	-4.2879	-4.2030	-4.2030	-3.4554
149	0.4249	0.4249	0.4249	-4.2879	-4.4177	-4.2879	-2.8931
151	0.4249	0.4249	0.4249	-4.3948	-4.4177	-4.3948	-4.4014
153	0.4249	0.4249	0.4249	-4.3948	-4.4177	-4.3948	-4.4014
155	0.4248	0.4249	0.4249	-4.5060	-4.4177	-4.4177	-3.6718
157	0.4248	0.4248	0.4248	-4.5060	-4.6343	-4.5060	-3.1104
159	0.4248	0.4248	0.4248	-4.6135	-4.6343	-4.6135	-4.6089
161	0.4248	0.4248	0.4248	-4.6135	-4.6343	-4.6135	-4.6089
163	0.4248	0.4248	0.4248	-4.7161	-4.6343	-4.6343	-3.8952
165	0.4248	0.4248	0.4248	-4.7161	-4.8474	-4.7161	-3.3290
167	0.4248	0.4248	0.4248	-4.8245	-4.8474	-4.8245	-4.8067
169	0.4248	0.4248	0.4248	-4.8245	-4.8474	-4.8245	-4.8067
171	0.4248	0.4248	0.4248	-4.9365	-4.8474	-4.8474	-4.1160
173	0.4248	0.4248	0.4248	-4.9365	-5.0693	-4.9365	-3.5429
175	0.4248	0.4248	0.4248	-5.0464	-5.0693	-5.0464	-5.0111
177	0.4248	0.4248	0.4248	-5.0464	-5.0693	-5.0464	-5.0111
179	0.4248	0.4248	0.4248	-5.1528	-5.0693	-5.0693	-4.3211
181	0.4248	0.4248	0.4248	-5.1528	-5.2844	-5.1528	-3.7585
183	0.4248	0.4248	0.4248	-5.2626	-5.2844	-5.2626	-5.2415
185	0.4248	0.4248	0.4248	-5.2626	-5.2844	-5.2626	-5.2415
187	0.4248	0.4248	0.4248	-5.3715	-5.2844	-5.2844	-4.5376
189	0.4248	0.4247	0.4248	-5.3715	-5.5039	-5.3715	-3.9769
191	0.4247	0.4247	0.4247	-5.4791	-5.5039	-5.4791	-5.4563
193	0.4247	0.4247	0.4247	-5.4791	-5.5039	-5.4791	-5.4563
195	0.4247	0.4247	0.4247	-5.5870	-5.5039	-5.5039	-4.7577
197	0.4247	0.4247	0.4247	-5.5870	-5.7151	-5.5870	-4.1898
199	0.4247	0.4247	0.4247	-5.6920	-5.7151	-5.6920	-5.6860
201	0.4247	0.4247	0.4247	-5.6920	-5.7151	-5.6920	-5.6860
203	0.4247	0.4247	0.4247	-5.8000	-5.7151	-5.7151	-4.9722
205	0.4247	0.4247	0.4247	-5.8000	-5.9301	-5.8000	-4.4043
207	0.4247	0.4247	0.4247	-5.9082	-5.9301	-5.9082	-5.8956
209	0.4247	0.4247	0.4247	-5.9082	-5.9301	-5.9082	-5.8956
211	0.4247	0.4247	0.4247	-6.0180	-5.9301	-5.9301	-5.1918
213	0.4247	0.4247	0.4247	-6.0180	-6.1494	-6.0180	-4.6227

215	0.4247	0.4247	0.4247	-6.1254	-6.1494	-6.1254	-6.0857
217	0.4247	0.4247	0.4247	-6.1254	-6.1494	-6.1254	-6.0857
219	0.4247	0.4247	0.4247	-6.2320	-6.1494	-6.1494	-5.3994
221	0.4247	0.4246	0.4247	-6.2320	-6.3581	-6.2320	-4.8343
223	0.4246	0.4246	0.4246	-6.3373	-6.3581	-6.3373	-6.3364

Table B.2. Performance Summary for I and Q filters where *TBW*= 0.150.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.3500	0.35	0.35	-1.1612	-1.4590	-1.1612	0.1755
23	0.3500	0.3500	0.3500	-1.4088	-1.4590	-1.4088	-1.1975
25	0.3500	0.3500	0.3500	-1.4088	-1.4590	-1.4088	-1.1975
27	0.3501	0.3500	0.3500	-1.6509	-1.4590	-1.4590	-0.6644
29	0.3501	0.3500	0.3501	-1.6509	-1.9402	-1.6509	-0.3175
31	0.3500	0.3500	0.3500	-1.8918	-1.9402	-1.8918	-1.6845
33	0.3500	0.3500	0.3500	-1.8918	-1.9402	-1.8918	-1.6845
35	0.3500	0.3500	0.3500	-2.1288	-1.9402	-1.9402	-1.1316
37	0.3500	0.3500	0.3500	-2.1288	-2.4096	-2.1288	-0.7900
39	0.3500	0.3500	0.3500	-2.3626	-2.4096	-2.3626	-2.1715
41	0.3500	0.3500	0.3500	-2.3626	-2.4096	-2.3626	-2.1715
43	0.3500	0.3500	0.3500	-2.5943	-2.4096	-2.4096	-1.6097
45	0.3500	0.3500	0.3500	-2.5943	-2.8714	-2.5943	-1.2559
47	0.3500	0.3500	0.3500	-2.8255	-2.8714	-2.8255	-2.6386
49	0.3500	0.3500	0.3500	-2.8255	-2.8714	-2.8255	-2.6386
51	0.3501	0.3500	0.3500	-3.0497	-2.8714	-2.8714	-2.0729
53	0.3501	0.3500	0.3501	-3.0497	-3.3259	-3.0497	-1.7159
55	0.3500	0.3500	0.3500	-3.2801	-3.3259	-3.2801	-3.0956
57	0.3500	0.3500	0.3500	-3.2801	-3.3259	-3.2801	-3.0956
59	0.3500	0.3500	0.3500	-3.5102	-3.3259	-3.3259	-2.5312
61	0.3500	0.3500	0.3500	-3.5102	-3.7807	-3.5102	-2.1688
63	0.3500	0.3500	0.3500	-3.7346	-3.7807	-3.7346	-3.5490
65	0.3500	0.3500	0.3500	-3.7346	-3.7807	-3.7346	-3.5490
67	0.3500	0.3500	0.3500	-3.9574	-3.7807	-3.7807	-2.9808
69	0.3500	0.3500	0.3500	-3.9574	-4.2265	-3.9574	-2.6225
71	0.3500	0.3500	0.3500	-4.1821	-4.2265	-4.1821	-4.0110
73	0.3500	0.3500	0.3500	-4.1821	-4.2265	-4.1821	-4.0110
75	0.3500	0.3500	0.3500	-4.4088	-4.2265	-4.2265	-3.4354
77	0.3500	0.3500	0.3500	-4.4088	-4.6768	-4.4088	-3.0690
79	0.3500	0.3500	0.3500	-4.6319	-4.6768	-4.6319	-4.4458
81	0.3500	0.3500	0.3500	-4.6319	-4.6768	-4.6319	-4.4458
83	0.3500	0.3500	0.3500	-4.8485	-4.6768	-4.6768	-3.8858

85	0.3500	0.3500	0.3500	-4.8485	-5.1206	-4.8485	-3.5146
87	0.3500	0.3500	0.3500	-5.0767	-5.1206	-5.0767	-4.8945
89	0.3500	0.3500	0.3500	-5.0767	-5.1206	-5.0767	-4.8945
91	0.3500	0.3500	0.3500	-5.3014	-5.1206	-5.1206	-4.3280
93	0.3500	0.3500	0.3500	-5.3014	-5.5579	-5.3014	-3.9605
95	0.3500	0.3500	0.3500	-5.5128	-5.5579	-5.5128	-5.3183
97	0.3500	0.3500	0.3500	-5.5128	-5.5579	-5.5128	-5.3183
99	0.3500	0.3500	0.3500	-5.7434	-5.5579	-5.5579	-4.7777
101	0.3500	0.3500	0.3500	-5.7434	-6.0082	-5.7434	-4.4008
103	0.3500	0.3500	0.3500	-5.9635	-6.0082	-5.9635	-5.7705
105	0.3500	0.3500	0.3500	-5.9635	-6.0082	-5.9635	-5.7705
107	0.3500	0.3500	0.3500	-6.1696	-6.0082	-6.0082	-5.2138
109	0.3500	0.3500	0.3500	-6.1696	-6.4461	-6.1696	-4.8457
111	0.3499	0.3500	0.3499	-6.4032	-6.4461	-6.4032	-6.2367
113	0.3499	0.3500	0.3499	-6.4032	-6.4461	-6.4032	-6.2367
115	0.3499	0.3500	0.3500	-6.6278	-6.4461	-6.4461	-5.6517
117	0.3499	0.3499	0.3499	-6.6278	-6.8947	-6.6278	-5.2863
119	0.3499	0.3499	0.3499	-6.8503	-6.8947	-6.8503	-6.6707
121	0.3499	0.3499	0.3499	-6.8503	-6.8947	-6.8503	-6.6707
123	0.3499	0.3499	0.3499	-7.0678	-6.8947	-6.8947	-6.0929
125	0.3499	0.3499	0.3499	-7.0678	-7.3336	-7.0678	-5.7259
127	0.3499	0.3499	0.3499	-7.2911	-7.3336	-7.2911	-7.1244
129	0.3499	0.3499	0.3499	-7.2911	-7.3336	-7.2911	-7.1244
131	0.3499	0.3499	0.3499	-7.5010	-7.3336	-7.3336	-6.5306
133	0.3499	0.3499	0.3499	-7.5010	-7.7670	-7.5010	-6.1641
135	0.3499	0.3499	0.3499	-7.7254	-7.7670	-7.7254	-7.5664

APPENDIX C - PERFORMANCE DATA FOR QUADRATURE DEMODULATION FILTERS DERIVED FROM A PROTOTYPE FILTER DESIGNED USING THE PARKS-McCLELLAN ALGORITHM

Table C.1 Performance summary for I and Q filters where *TBW*=0.075.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.4004	0.4536	0.4255	-0.7965	-0.4751	-0.4751	0.7551
23	0.3745	0.4546	0.4258	-1.1124	-0.4589	-0.4589	0.7375
25	0.4394	0.3861	0.4394	-0.4857	-1.5522	-0.4857	0.7071
27	0.4388	0.4263	0.4263	-0.9515	-0.5893	-0.5893	0.6088
29	0.4040	0.4448	0.4232	-1.0080	-0.7194	-0.7194	0.4846
31	0.4024	0.4457	0.4251	-1.1559	-0.7143	-0.7143	0.4649
33	0.4381	0.3855	0.4326	-0.7405	-1.8824	-0.7405	0.4357
35	0.4289	0.4306	0.4306	-1.1563	-0.8330	-0.8330	0.3529
37	0.4223	0.4386	0.4243	-1.0910	-0.9716	-0.9716	0.2279
39	0.4151	0.4391	0.4262	-1.2661	-0.9600	-0.9600	0.2090
41	0.4364	0.3998	0.4266	-0.9856	-1.9522	-0.9856	0.1822
43	0.4209	0.4310	0.4310	-1.4286	-1.0704	-1.0704	0.1025
45	0.4211	0.4343	0.4264	-1.3455	-1.2114	-1.2114	-0.0159
47	0.4218	0.4342	0.4276	-1.4052	-1.2007	-1.2007	-0.0366
49	0.4346	0.4083	0.4251	-1.2265	-1.9744	-1.2265	-0.0624
51	0.4192	0.4316	0.4279	-1.6633	-1.2995	-1.2995	-0.1377
53	0.4279	0.4304	0.4279	-1.4486	-1.4483	-1.4385	-0.2512
55	0.4256	0.4302	0.4286	-1.5629	-1.4364	-1.4364	-0.2759
57	0.4326	0.4152	0.4253	-1.4604	-2.0041	-1.4604	-0.2999
59	0.4187	0.4312	0.4255	-1.9271	-1.5356	-1.5356	-0.3717
61	0.4270	0.4273	0.4273	-1.7205	-1.6868	-1.6671	-0.4839
63	0.4275	0.4274	0.4274	-1.7429	-1.6694	-1.6694	-0.5095
65	0.4307	0.4202	0.4261	-1.6921	-2.0624	-1.6921	-0.5334
67	0.4196	0.4305	0.4250	-2.1703	-1.7651	-1.7651	-0.6035
69	0.4290	0.4255	0.4262	-1.8700	-1.9189	-1.8700	-0.7128
71	0.4284	0.4257	0.4257	-1.9406	-1.8980	-1.8980	-0.7390
73	0.4288	0.4234	0.4268	-1.9202	-2.1637	-1.9202	-0.7634
75	0.4211	0.4296	0.4252	-2.4166	-1.9886	-1.9886	-0.8314
77	0.4285	0.4245	0.4245	-2.1242	-2.1509	-2.1221	-0.9386

79	0.4286	0.4250	0.4250	-2.1600	-2.1252	-2.1252	-0.9666
81	0.4271	0.4254	0.4271	-2.1472	-2.2989	-2.1472	-0.9899
83	0.4227	0.4285	0.4258	-2.5473	-2.2149	-2.2149	-1.0557
85	0.4285	0.4243	0.4250	-2.3470	-2.3850	-2.3470	-1.1641
87	0.4283	0.4251	0.4251	-2.4020	-2.3506	-2.3506	-1.1923
89	0.4259	0.4265	0.4259	-2.3715	-2.4647	-2.3715	-1.2144
91	0.4239	0.4274	0.4263	-2.7074	-2.4394	-2.4394	-1.2810
93	0.4279	0.4247	0.4252	-2.5444	-2.6134	-2.5444	-1.3856
95	0.4278	0.4255	0.4255	-2.6036	-2.5745	-2.5744	-1.4164
97	0.4252	0.4270	0.4252	-2.5945	-2.6549	-2.5945	-1.4376
99	0.4249	0.4263	0.4263	-2.8633	-2.6610	-2.6610	-1.5019
101	0.4272	0.4252	0.4257	-2.7823	-2.8405	-2.7823	-1.6049
103	0.4268	0.4259	0.4259	-2.8142	-2.7980	-2.7980	-1.6402
105	0.4250	0.4272	0.4250	-2.8157	-2.8654	-2.8157	-1.6591
107	0.4259	0.4256	0.4256	-3.0049	-2.8805	-2.8805	-1.7231
109	0.4263	0.4257	0.4260	-2.9917	-3.0698	-2.9917	-1.8241
111	0.4259	0.4263	0.4261	-3.0418	-3.0193	-3.0193	-1.8612
113	0.4251	0.4270	0.4251	-3.0364	-3.1017	-3.0364	-1.8799
115	0.4263	0.4251	0.4251	-3.1825	-3.0996	-3.0996	-1.9420
117	0.4259	0.4260	0.4259	-3.2020	-3.2678	-3.2020	-2.0440
119	0.4253	0.4265	0.4255	-3.2545	-3.2396	-3.2390	-2.0812
121	0.4254	0.4266	0.4254	-3.2576	-3.3078	-3.2576	-2.1008
123	0.4265	0.4249	0.4249	-3.3826	-3.3181	-3.3181	-2.1609
125	0.4251	0.4261	0.4251	-3.4306	-3.4700	-3.4303	-2.2625
127	0.4250	0.4265	0.4250	-3.4692	-3.4585	-3.4585	-2.3004
129	0.4258	0.4261	0.4258	-3.4752	-3.5059	-3.4752	-2.3187
131	0.4264	0.4251	0.4251	-3.6125	-3.5372	-3.5372	-2.3792
133	0.4249	0.4262	0.4249	-3.6448	-3.6958	-3.6398	-2.4796
135	0.4249	0.4264	0.4249	-3.6942	-3.6796	-3.6796	-2.5216
137	0.4260	0.4255	0.4257	-3.6936	-3.7202	-3.6936	-2.5367
139	0.4262	0.4253	0.4253	-3.8378	-3.7561	-3.7561	-2.5989
141	0.4249	0.4261	0.4249	-3.8547	-3.9424	-3.8547	-2.6968
143	0.4250	0.4263	0.4250	-3.9048	-3.8989	-3.8989	-2.7403
145	0.4261	0.4249	0.4252	-3.9133	-3.9520	-3.9133	-2.7563

147	0.4259	0.4256	0.4256	-4.0185	-3.9710	-3.9710	-2.8134
149	0.4250	0.4260	0.4250	-4.0725	-4.1570	-4.0725	-2.9127
151	0.4252	0.4260	0.4252	-4.1222	-4.1144	-4.1144	-2.9562
153	0.4261	0.4248	0.4249	-4.1307	-4.1498	-4.1307	-2.9735
155	0.4254	0.4258	0.4254	-4.2204	-4.1862	-4.1862	-3.0287
157	0.4252	0.4259	0.4252	-4.2911	-4.3614	-4.2911	-3.1266
159	0.4255	0.4257	0.4255	-4.3365	-4.3289	-4.3289	-3.1726
161	0.4261	0.4248	0.4248	-4.3451	-4.3604	-4.3451	-3.1873
163	0.4248	0.4259	0.4250	-4.4498	-4.4032	-4.4032	-3.2459
165	0.4255	0.4256	0.4255	-4.4987	-4.5681	-4.4987	-3.3405
167	0.4257	0.4252	0.4252	-4.5547	-4.5491	-4.5486	-3.3905
169	0.4260	0.4249	0.4249	-4.5605	-4.5833	-4.5605	-3.4042
171	0.4246	0.4260	0.4248	-4.6569	-4.6152	-4.6152	-3.4589
173	0.4257	0.4251	0.4252	-4.7124	-4.7587	-4.7124	-3.5557
175	0.4259	0.4249	0.4249	-4.7658	-4.7630	-4.7630	-3.6049
177	0.4258	0.4251	0.4251	-4.7789	-4.7897	-4.7789	-3.6208
179	0.4247	0.4259	0.4248	-4.8515	-4.8304	-4.8304	-3.6726
181	0.4258	0.4247	0.4249	-4.9271	-4.9818	-4.9271	-3.7695
183	0.4259	0.4248	0.4248	-4.9846	-4.9782	-4.9782	-3.8208
185	0.4254	0.4253	0.4253	-4.9908	-5.0048	-4.9908	-3.8337
187	0.4248	0.4258	0.4249	-5.0688	-5.0456	-5.0456	-3.8885
189	0.4258	0.4244	0.4247	-5.1441	-5.2095	-5.1441	-3.9824
191	0.4258	0.4248	0.4248	-5.1985	-5.1964	-5.1964	-4.0378
193	0.4251	0.4254	0.4251	-5.2065	-5.2175	-5.2060	-4.0487
195	0.4250	0.4256	0.4250	-5.2924	-5.2592	-5.2592	-4.1015
197	0.4258	0.4246	0.4248	-5.3553	-5.3919	-5.3549	-4.1962
199	0.4256	0.4249	0.4249	-5.4116	-5.4088	-5.4073	-4.2506
201	0.4248	0.4256	0.4248	-5.4229	-5.4313	-5.4229	-4.2649
203	0.4252	0.4252	0.4252	-5.4889	-5.4719	-5.4719	-4.3150
205	0.4256	0.4248	0.4248	-5.5661	-5.5979	-5.5661	-4.4085
207	0.4253	0.4251	0.4251	-5.6257	-5.6236	-5.6236	-4.4666
209	0.4247	0.4256	0.4247	-5.6345	-5.6478	-5.6345	-4.4765
211	0.4253	0.4249	0.4249	-5.7039	-5.6878	-5.6878	-4.5307
213	0.4253	0.4250	0.4250	-5.7795	-5.8220	-5.7795	-4.6212

215	0.4250	0.4252	0.4250	-5.8421	-5.8435	-5.8416	-4.6833
217	0.4247	0.4255	0.4247	-5.8495	-5.8561	-5.8495	-4.6912
219	0.4253	0.4247	0.4247	-5.9178	-5.8997	-5.8994	-4.7428
221	0.4250	0.4251	0.4250	-5.9930	-6.0253	-5.9917	-4.8342
223	0.4248	0.4253	0.4248	-6.0571	-6.0529	-6.0529	-4.8948

Table C.2. Performance summary for I and Q filters where *TBW*=0.150.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
23	0.3354	0.3683	0.3524	-1.4692	-1.1652	-1.1652	0.0156
25	0.3698	0.3128	0.3502	-1.1822	-1.9594	-1.1822	-0.0179
27	0.3198	0.3632	0.3552	-2.2158	-1.3136	-1.3136	-0.1496
29	0.3530	0.3532	0.3530	-1.5871	-1.7154	-1.5871	-0.3643
31	0.3533	0.3556	0.3556	-1.7390	-1.6399	-1.6328	-0.4663
33	0.3620	0.3393	0.3517	-1.6569	-2.0412	-1.6569	-0.4973
35	0.3302	0.3612	0.3501	-2.5815	-1.7765	-1.7765	-0.6182
37	0.3579	0.3434	0.3515	-1.9768	-2.2460	-1.9768	-0.8192
39	0.3571	0.3495	0.3495	-2.1384	-2.1061	-2.1061	-0.9356
41	0.3548	0.3500	0.3543	-2.1209	-2.2885	-2.1209	-0.9627
43	0.3429	0.3570	0.3516	-2.6584	-2.2301	-2.2301	-1.0735
45	0.3565	0.3457	0.3499	-2.4302	-2.7553	-2.4302	-1.2679
47	0.3554	0.3512	0.3512	-2.6027	-2.5682	-2.5682	-1.3999
49	0.3506	0.3538	0.3506	-2.5795	-2.6407	-2.5764	-1.4202
51	0.3501	0.3527	0.3527	-2.8631	-2.6789	-2.6789	-1.5227
53	0.3540	0.3480	0.3515	-2.8741	-3.1573	-2.8741	-1.7089
55	0.3510	0.3528	0.3514	-3.0442	-3.0256	-3.0256	-1.8558
57	0.3501	0.3541	0.3501	-3.0286	-3.0880	-3.0286	-1.8697
59	0.3528	0.3502	0.3502	-3.1957	-3.1217	-3.1217	-1.9640
61	0.3512	0.3502	0.3512	-3.3096	-3.5042	-3.3096	-2.1456
63	0.3499	0.3534	0.3500	-3.4801	-3.4749	-3.4745	-2.3068
65	0.3513	0.3525	0.3513	-3.4789	-3.5117	-3.4789	-2.3199
67	0.3530	0.3502	0.3502	-3.6318	-3.5647	-3.5647	-2.4074
69	0.3500	0.3520	0.3500	-3.7431	-3.8253	-3.7424	-2.5822
71	0.3508	0.3520	0.3509	-3.9290	-3.9184	-3.9184	-2.7587
73	0.3521	0.3502	0.3506	-3.9249	-3.9598	-3.9249	-2.7656
75	0.3518	0.3512	0.3512	-4.0474	-4.0030	-4.0030	-2.8459
77	0.3503	0.3521	0.3503	-4.1712	-4.2618	-4.1712	-3.0129
79	0.3519	0.3502	0.3503	-4.3644	-4.3696	-4.3644	-3.2037
81	0.3521	0.3498	0.3500	-4.3699	-4.3845	-4.3699	-3.2095
83	0.3500	0.3518	0.3504	-4.4798	-4.4393	-4.4393	-3.2825
85	0.3511	0.3511	0.3511	-4.6001	-4.6562	-4.6001	-3.4424

87	0.3517	0.3502	0.3502	-4.8150	-4.8095	-4.8061	-3.6476
89	0.3516	0.3504	0.3504	-4.8078	-4.8253	-4.8049	-3.6483
91	0.3498	0.3518	0.3500	-4.8955	-4.8736	-4.8736	-3.7158
93	0.3516	0.3498	0.3501	-5.0260	-5.0811	-5.0260	-3.8711
95	0.3505	0.3509	0.3505	-5.2556	-5.2600	-5.2460	-4.0857
97	0.3505	0.3509	0.3505	-5.2506	-5.2655	-5.2493	-4.0905
99	0.3503	0.3512	0.3503	-5.3363	-5.3082	-5.3082	-4.1496
101	0.3515	0.3498	0.3500	-5.4554	-5.4851	-5.4554	-4.2978
103	0.3497	0.3512	0.3500	-5.7290	-5.6671	-5.6671	-4.5087
105	0.3500	0.3512	0.3500	-5.6924	-5.6867	-5.6858	-4.5329
107	0.3507	0.3504	0.3504	-5.7568	-5.7430	-5.7430	-4.5853
109	0.3508	0.3504	0.3504	-5.8842	-5.9234	-5.8842	-4.7268
111	0.3498	0.3510	0.3500	-6.1420	-6.0930	-6.0930	-4.9351
113	0.3501	0.3510	0.3501	-6.1271	-6.1346	-6.1271	-4.9685
115	0.3510	0.3500	0.3500	-6.1854	-6.1765	-6.1765	-5.0175
117	0.3502	0.3507	0.3502	-6.3124	-6.3304	-6.3124	-5.1563
119	0.3503	0.3505	0.3504	-6.5600	-6.5166	-6.5166	-5.3581
121	0.3504	0.3503	0.3504	-6.5677	-6.5716	-6.5677	-5.4075
123	0.3508	0.3500	0.3500	-6.6202	-6.6085	-6.6074	-5.4507
125	0.3499	0.3507	0.3499	-6.7359	-6.7528	-6.7354	-5.5794
127	0.3506	0.3500	0.3500	-6.9561	-6.9382	-6.9382	-5.7803
129	0.3506	0.3500	0.3500	-7.0005	-7.0160	-7.0005	-5.8504
131	0.3503	0.3503	0.3503	-7.0530	-7.0420	-7.0412	-5.8836
133	0.3500	0.3506	0.3500	-7.1620	-7.1830	-7.1620	-6.0052
135	0.3505	0.3499	0.3499	-7.4021	-7.3595	-7.3595	-6.1994

APPENDIX D - PERFORMANCE DATA FOR QUADRATURE DEMODULATION FILTERS DERIVED FROM A PROTOTYPE FILTER DESIGNED USING THE PARKS-McCLELLAN ALGORITHM (MINTZER TECHNIQUE)

Table D.1 Performance summary for I and Q filters where $TBW = 0.075$.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.4004	0.4544	0.4248	-0.7965	-0.4873	-0.4873	0.8016
23	0.3745	0.4543	0.4256	-1.1124	-0.4618	-0.4618	0.7344
25	0.4394	0.4174	0.4388	-0.4857	-0.8274	-0.4857	0.4707
27	0.4388	0.4112	0.4131	-0.9515	-0.8854	-0.8640	0.4453
29	0.4040	0.4447	0.4226	-1.0080	-0.7265	-0.7265	0.5769
31	0.4024	0.4455	0.4249	-1.1559	-0.7173	-0.7173	0.4632
33	0.4381	0.4209	0.4335	-0.7405	-1.0588	-0.7405	0.1935
35	0.4289	0.4183	0.4183	-1.1563	-1.0836	-1.0836	0.2086
37	0.4223	0.4385	0.4243	-1.0910	-0.9759	-0.9759	0.3561
39	0.4151	0.4391	0.4261	-1.2661	-0.9639	-0.9639	0.2050
41	0.4364	0.4231	0.4311	-0.9856	-1.2792	-0.9856	-0.0692
43	0.4209	0.4223	0.4223	-1.4286	-1.2817	-1.2817	-0.0252
45	0.4211	0.4336	0.4262	-1.3455	-1.2227	-1.2227	0.1244
47	0.4218	0.4342	0.4275	-1.4052	-1.2052	-1.2052	-0.0409
49	0.4346	0.4245	0.4299	-1.2265	-1.4947	-1.2265	-0.3211
51	0.4192	0.4252	0.4220	-1.6633	-1.4682	-1.4682	-0.2545
53	0.4279	0.4295	0.4279	-1.4486	-1.4587	-1.4420	-0.0947
55	0.4256	0.4303	0.4285	-1.5629	-1.4410	-1.4410	-0.2799
57	0.4326	0.4254	0.4292	-1.4604	-1.7086	-1.4604	-0.5629
59	0.4187	0.4269	0.4220	-1.9271	-1.6606	-1.6606	-0.4803
61	0.4270	0.4266	0.4266	-1.7205	-1.6965	-1.6702	-0.3264
63	0.4275	0.4274	0.4274	-1.7429	-1.6751	-1.6751	-0.5149
65	0.4307	0.4259	0.4286	-1.6921	-1.9262	-1.6921	-0.8004
67	0.4196	0.4278	0.4229	-2.1703	-1.8630	-1.8630	-0.7039
69	0.4290	0.4249	0.4258	-1.8700	-1.9339	-1.8700	-0.5524
71	0.4284	0.4257	0.4257	-1.9406	-1.9043	-1.9015	-0.7415
73	0.4288	0.4260	0.4281	-1.9202	-2.1500	-1.9202	-1.0337
75	0.4211	0.4280	0.4241	-2.4166	-2.0800	-2.0800	-0.9327
77	0.4285	0.4242	0.4244	-2.1242	-2.1662	-2.1242	-0.7817

79	0.4286	0.4250	0.4250	-2.1600	-2.1321	-2.1321	-0.9682
81	0.4271	0.4260	0.4271	-2.1472	-2.3785	-2.1472	-1.2648
83	0.4227	0.4279	0.4251	-2.5473	-2.3139	-2.3139	-1.1513
85	0.4285	0.4244	0.4251	-2.3470	-2.4004	-2.3470	-1.0083
87	0.4283	0.4250	0.4250	-2.4020	-2.3586	-2.3586	-1.1947
89	0.4259	0.4258	0.4259	-2.3715	-2.6079	-2.3715	-1.4915
91	0.4239	0.4274	0.4259	-2.7074	-2.5210	-2.5210	-1.3730
93	0.4279	0.4248	0.4255	-2.5444	-2.6341	-2.5444	-1.2335
95	0.4278	0.4253	0.4254	-2.6036	-2.5835	-2.5789	-1.4181
97	0.4252	0.4256	0.4252	-2.5945	-2.8273	-2.5945	-1.7138
99	0.4249	0.4267	0.4263	-2.8633	-2.7338	-2.7150	-1.5906
101	0.4272	0.4253	0.4259	-2.7823	-2.8626	-2.7823	-1.4550
103	0.4268	0.4258	0.4258	-2.8142	-2.8072	-2.8060	-1.6421
105	0.4250	0.4253	0.4250	-2.8157	-3.0479	-2.8157	-1.9423
107	0.4259	0.4260	0.4260	-3.0049	-2.9557	-2.9234	-1.8055
109	0.4263	0.4258	0.4262	-2.9917	-3.0791	-2.9917	-1.6772
111	0.4259	0.4262	0.4261	-3.0418	-3.0297	-3.0276	-1.8632
113	0.4251	0.4251	0.4251	-3.0364	-3.2728	-3.0364	-2.1652
115	0.4263	0.4255	0.4255	-3.1825	-3.1652	-3.1388	-2.0234
117	0.4259	0.4261	0.4259	-3.2020	-3.2714	-3.2020	-1.8998
119	0.4253	0.4264	0.4254	-3.2545	-3.2510	-3.2446	-2.0840
121	0.4254	0.4250	0.4254	-3.2576	-3.4931	-3.2576	-2.3834
123	0.4265	0.4251	0.4251	-3.3826	-3.3801	-3.3668	-2.2419
125	0.4251	0.4261	0.4251	-3.4306	-3.4854	-3.4306	-2.1216
127	0.4250	0.4265	0.4250	-3.4692	-3.4642	-3.4637	-2.3022
129	0.4258	0.4249	0.4258	-3.4752	-3.7167	-3.4752	-2.6057
131	0.4264	0.4249	0.4250	-3.6125	-3.5986	-3.5887	-2.4542
133	0.4249	0.4261	0.4249	-3.6448	-3.7224	-3.6448	-2.3417
135	0.4249	0.4264	0.4249	-3.6942	-3.6931	-3.6863	-2.5243
137	0.4260	0.4249	0.4258	-3.6936	-3.9366	-3.6936	-2.8267
139	0.4262	0.4249	0.4251	-3.8378	-3.8127	-3.7885	-2.6720
141	0.4249	0.4261	0.4249	-3.8547	-3.9424	-3.8547	-2.5628
143	0.4250	0.4262	0.4250	-3.9048	-3.9135	-3.9048	-2.7423
145	0.4261	0.4249	0.4257	-3.9133	-4.1581	-3.9133	-3.0428

147	0.4259	0.4250	0.4252	-4.0185	-4.0266	-3.9960	-2.8857
149	0.4250	0.4260	0.4250	-4.0725	-4.1719	-4.0725	-2.7821
151	0.4252	0.4260	0.4252	-4.1222	-4.1299	-4.1222	-2.9575
153	0.4261	0.4249	0.4256	-4.1307	-4.3785	-4.1307	-3.2612
155	0.4254	0.4253	0.4254	-4.2204	-4.2395	-4.2204	-3.0969
157	0.4252	0.4258	0.4252	-4.2911	-4.3698	-4.2906	-2.9999
159	0.4255	0.4258	0.4255	-4.3365	-4.3413	-4.3336	-3.1781
161	0.4261	0.4249	0.4256	-4.3451	-4.5943	-4.3451	-3.4790
163	0.4248	0.4255	0.4249	-4.4498	-4.4528	-4.4333	-3.3099
165	0.4255	0.4253	0.4254	-4.4987	-4.5570	-4.4987	-3.2168
167	0.4257	0.4253	0.4254	-4.5547	-4.5652	-4.5547	-3.3957
169	0.4260	0.4249	0.4255	-4.5605	-4.8146	-4.5605	-3.6934
171	0.4246	0.4256	0.4247	-4.6569	-4.6641	-4.6423	-3.5224
173	0.4257	0.4248	0.4250	-4.7124	-4.7686	-4.7124	-3.4358
175	0.4259	0.4250	0.4250	-4.7658	-4.7811	-4.7658	-3.6101
177	0.4258	0.4249	0.4255	-4.7789	-5.0266	-4.7789	-3.9061
179	0.4247	0.4257	0.4247	-4.8515	-4.8754	-4.8515	-3.7323
181	0.4258	0.4245	0.4248	-4.9271	-5.0028	-4.9271	-3.6549
183	0.4259	0.4248	0.4248	-4.9846	-4.9966	-4.9846	-3.8255
185	0.4254	0.4249	0.4254	-4.9908	-5.2478	-4.9908	-4.1243
187	0.4248	0.4257	0.4248	-5.0688	-5.0912	-5.0688	-3.9433
189	0.4258	0.4246	0.4248	-5.1441	-5.1883	-5.1441	-3.8719
191	0.4258	0.4247	0.4248	-5.1985	-5.2077	-5.1985	-4.0459
193	0.4251	0.4248	0.4251	-5.2065	-5.4677	-5.2065	-4.3434
195	0.4250	0.4256	0.4250	-5.2924	-5.3015	-5.2818	-4.1558
197	0.4258	0.4247	0.4249	-5.3553	-5.3926	-5.3553	-4.0895
199	0.4256	0.4248	0.4248	-5.4116	-5.4292	-5.4116	-4.2596
201	0.4248	0.4248	0.4248	-5.4229	-5.6849	-5.4229	-4.5588
203	0.4252	0.4254	0.4252	-5.4889	-5.5125	-5.4889	-4.3649
205	0.4256	0.4249	0.4250	-5.5661	-5.6144	-5.5661	-4.3082
207	0.4253	0.4249	0.4249	-5.6257	-5.6464	-5.6257	-4.4766
209	0.4247	0.4247	0.4247	-5.6345	-5.9006	-5.6345	-4.7740
211	0.4253	0.4251	0.4252	-5.7039	-5.7281	-5.7039	-4.5784
213	0.4253	0.4251	0.4251	-5.7795	-5.8208	-5.7795	-4.5257

215	0.4250	0.4251	0.4250	-5.8421	-5.8584	-5.8421	-4.6963
217	0.4247	0.4247	0.4247	-5.8495	-6.1201	-5.8495	-4.9872
219	0.4253	0.4249	0.4249	-5.9178	-5.9384	-5.9178	-4.7882
221	0.4250	0.4252	0.4250	-5.9930	-6.0255	-5.9930	-4.7439
223	0.4248	0.4252	0.4248	-6.0571	-6.0759	-6.0571	-4.9082

Table D.2. Performance summary for I and Q filters where $TBW = 0.150$.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.3496	0.3664	0.3496	-1.1505	-1.2958	-1.1505	0.2293
23	0.3354	0.3689	0.3522	-1.4692	-1.1753	-1.1753	0.0055
25	0.3698	0.3496	0.3607	-1.1822	-1.4587	-1.1822	-0.3094
27	0.3198	0.3539	0.3480	-2.2158	-1.4132	-1.4132	-0.2339
29	0.3530	0.3475	0.3520	-1.5871	-1.7707	-1.5871	-0.2422
31	0.3533	0.3562	0.3562	-1.7390	-1.6548	-1.6419	-0.4773
33	0.3620	0.3510	0.3572	-1.6569	-1.9202	-1.6569	-0.7971
35	0.3302	0.3573	0.3476	-2.5815	-1.8366	-1.8366	-0.6787
37	0.3579	0.3432	0.3507	-1.9768	-2.2920	-1.9768	-0.7108
39	0.3571	0.3497	0.3497	-2.1384	-2.1250	-2.1250	-0.9446
41	0.3548	0.3509	0.3548	-2.1209	-2.3893	-2.1209	-1.2627
43	0.3429	0.3565	0.3509	-2.6584	-2.2827	-2.2827	-1.1222
45	0.3565	0.3459	0.3507	-2.4302	-2.8058	-2.4302	-1.1793
47	0.3554	0.3502	0.3506	-2.6027	-2.5921	-2.5788	-1.4145
49	0.3506	0.3504	0.3506	-2.5795	-2.8582	-2.5795	-1.7157
51	0.3501	0.3534	0.3528	-2.8631	-2.7211	-2.7211	-1.5613
53	0.3540	0.3485	0.3525	-2.8741	-3.1508	-2.8741	-1.6407
55	0.3510	0.3518	0.3511	-3.0442	-3.0513	-3.0400	-1.8818
57	0.3501	0.3500	0.3501	-3.0286	-3.3175	-3.0286	-2.1587
59	0.3528	0.3509	0.3509	-3.1957	-3.1516	-3.1368	-1.9903
61	0.3512	0.3516	0.3512	-3.3096	-3.4385	-3.3096	-2.1017
63	0.3499	0.3526	0.3499	-3.4801	-3.5017	-3.4801	-2.3417
65	0.3513	0.3500	0.3513	-3.4789	-3.7761	-3.4789	-2.5978
67	0.3530	0.3500	0.3500	-3.6318	-3.5872	-3.5872	-2.4265
69	0.3500	0.3522	0.3500	-3.7431	-3.8325	-3.7431	-2.5653
71	0.3508	0.3520	0.3508	-3.9290	-3.9657	-3.9290	-2.7979
73	0.3521	0.3500	0.3515	-3.9249	-4.2162	-3.9249	-3.0288
75	0.3518	0.3506	0.3507	-4.0474	-4.0175	-4.0113	-2.8574
77	0.3503	0.3515	0.3503	-4.1712	-4.2704	-4.1712	-3.0249
79	0.3519	0.3511	0.3514	-4.3644	-4.4216	-4.3644	-3.2465
81	0.3521	0.3500	0.3512	-4.3699	-4.6604	-4.3699	-3.4600
83	0.3500	0.3514	0.3503	-4.4798	-4.4477	-4.4477	-3.2886

85	0.3511	0.3501	0.3505	-4.6001	-4.6782	-4.6001	-3.4842
87	0.3517	0.3501	0.3508	-4.8150	-4.9140	-4.8150	-3.6934
89	0.3516	0.3500	0.3510	-4.8078	-5.0992	-4.8078	-3.8798
91	0.3498	0.3517	0.3500	-4.8955	-4.8762	-4.8762	-3.7187
93	0.3516	0.3498	0.3502	-5.0260	-5.1073	-5.0260	-3.9376
95	0.3505	0.3500	0.3505	-5.2556	-5.5251	-5.2556	-4.3165
97	0.3505	0.3500	0.3505	-5.2506	-5.5368	-5.2506	-4.3056
99	0.3503	0.3514	0.3503	-5.3363	-5.3078	-5.3078	-4.1496
101	0.3515	0.3502	0.3504	-5.4554	-5.5441	-5.4554	-4.3752
103	0.3497	0.3500	0.3497	-5.7290	-5.9315	-5.7290	-4.7477
105	0.3500	0.3500	0.3500	-5.6924	-5.9762	-5.6924	-4.7302
107	0.3507	0.3506	0.3506	-5.7568	-5.7434	-5.7434	-4.5855
109	0.3508	0.3505	0.3507	-5.8842	-5.9642	-5.8842	-4.8188
111	0.3498	0.3501	0.3498	-6.1420	-6.3335	-6.1420	-5.1812
113	0.3501	0.3500	0.3501	-6.1271	-6.4123	-6.1271	-5.1463
115	0.3510	0.3500	0.3500	-6.1854	-6.1797	-6.1785	-5.0222
117	0.3502	0.3506	0.3502	-6.3124	-6.3929	-6.3124	-5.2542
119	0.3503	0.3501	0.3503	-6.5600	-6.7369	-6.5600	-5.6079
121	0.3504	0.3499	0.3504	-6.5677	-6.8566	-6.5677	-5.5695
123	0.3508	0.3500	0.3500	-6.6202	-6.6161	-6.6104	-5.4537
125	0.3499	0.3504	0.3499	-6.7359	-6.8335	-6.7359	-5.6949
127	0.3506	0.3501	0.3503	-6.9561	-7.1400	-6.9561	-6.0200
129	0.3506	0.3499	0.3503	-7.0005	-7.2914	-7.0005	-5.9974
131	0.3503	0.3502	0.3502	-7.0530	-7.0606	-7.0506	-5.8843
133	0.3500	0.3501	0.3500	-7.1620	-7.2634	-7.1620	-6.1347
135	0.3505	0.3500	0.3501	-7.4021	-7.5396	-7.4021	-6.4263

APPENDIX E - PERFORMANCE DATA FOR QUADRATURE DEMODULATION FILTERS SEPARATELY DESIGNED USING THE KAISER WINDOW

Table E.1 Performance summary for I and Q filters where *TBW*= 0.075.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.3787	0.4159	0.4159	-1.1421	-0.8369	-0.7643	0.5390
23	0.4208	0.4159	0.4208	-0.7317	-0.8369	-0.7317	0.1063
25	0.4208	0.4159	0.4208	-0.7317	-0.8369	-0.7317	0.1063
27	0.4121	0.4159	0.4159	-1.0979	-0.8369	-0.8009	0.4326
29	0.4121	0.4251	0.4121	-1.0979	-0.9573	-0.8996	0.2932
31	0.4257	0.4251	0.4251	-0.8977	-0.9573	-0.8962	-0.2924
33	0.4257	0.4195	0.4199	-0.8977	-1.0289	-0.8977	-0.0652
35	0.4155	0.4195	0.4155	-1.2753	-1.0289	-1.0129	0.1077
37	0.4155	0.4225	0.4225	-1.2753	-1.2485	-1.1258	-0.0037
39	0.4248	0.4225	0.4248	-1.1833	-1.2485	-1.1767	-0.5141
41	0.4248	0.4212	0.4248	-1.1833	-1.2681	-1.1833	-0.4618
43	0.4175	0.4212	0.4212	-1.4686	-1.2681	-1.2681	-0.2026
45	0.4175	0.4209	0.4175	-1.4686	-1.4989	-1.3998	-0.2855
47	0.4238	0.4209	0.4209	-1.4272	-1.4989	-1.4272	-0.6822
49	0.4238	0.4221	0.4221	-1.4272	-1.4807	-1.4252	-0.8182
51	0.4191	0.4221	0.4191	-1.6560	-1.4807	-1.4770	-0.4983
53	0.4191	0.4203	0.4203	-1.6560	-1.7422	-1.5840	-0.5406
55	0.4230	0.4203	0.4222	-1.6827	-1.7422	-1.6628	-0.9142
57	0.4230	0.4226	0.4230	-1.6827	-1.7035	-1.6603	-1.1468
59	0.4205	0.4226	0.4226	-1.8422	-1.7035	-1.6993	-0.7804
61	0.4205	0.4204	0.4205	-1.8422	-1.9561	-1.8329	-0.8041
63	0.4227	0.4204	0.4204	-1.9181	-1.9561	-1.9130	-1.0403
65	0.4227	0.4231	0.4230	-1.9181	-1.9150	-1.9026	-1.4669
67	0.4217	0.4231	0.4217	-2.0236	-1.9150	-1.9117	-1.0496
69	0.4217	0.4211	0.4211	-2.0236	-2.1508	-1.9970	-1.0380
71	0.4227	0.4211	0.4222	-2.1443	-2.1508	-2.0887	-1.2902
73	0.4227	0.4236	0.4227	-2.1443	-2.1265	-2.1144	-1.5011
75	0.4228	0.4236	0.4236	-2.2036	-2.1265	-2.1051	-1.3093
77	0.4228	0.4221	0.4228	-2.2036	-2.3275	-2.2036	-1.2885

79	0.4230	0.4221	0.4221	-2.3526	-2.3275	-2.3268	-1.5411
81	0.4230	0.4238	0.4233	-2.3526	-2.3389	-2.3299	-2.0358
83	0.4235	0.4238	0.4235	-2.3898	-2.3389	-2.3277	-1.5715
85	0.4235	0.4228	0.4228	-2.3898	-2.5049	-2.3846	-1.5397
87	0.4232	0.4228	0.4231	-2.5592	-2.5049	-2.4748	-1.6762
89	0.4232	0.4241	0.4233	-2.5592	-2.5446	-2.5446	-1.8660
91	0.4242	0.4241	0.4241	-2.5761	-2.5446	-2.5125	-1.8236
93	0.4242	0.4236	0.4242	-2.5761	-2.6762	-2.5750	-1.7728
95	0.4237	0.4236	0.4236	-2.7490	-2.6762	-2.6762	-1.9407
97	0.4237	0.4244	0.4239	-2.7490	-2.7492	-2.7289	-2.2055
99	0.4246	0.4244	0.4245	-2.7677	-2.7492	-2.7290	-2.0772
101	0.4246	0.4242	0.4242	-2.7677	-2.8505	-2.7676	-2.0129
103	0.4241	0.4242	0.4241	-2.9369	-2.8505	-2.8469	-2.0912
105	0.4241	0.4245	0.4241	-2.9369	-2.9528	-2.9309	-2.4205
107	0.4249	0.4245	0.4245	-2.9643	-2.9528	-2.9253	-2.2472
109	0.4249	0.4246	0.4249	-2.9643	-3.0311	-2.9517	-2.2524
111	0.4244	0.4246	0.4246	-3.1218	-3.0311	-3.0311	-2.3457
113	0.4244	0.4247	0.4244	-3.1218	-3.1605	-3.1181	-2.5872
115	0.4250	0.4247	0.4249	-3.1694	-3.1605	-3.1359	-2.5880
117	0.4250	0.4249	0.4249	-3.1694	-3.2214	-3.1666	-2.4917
119	0.4247	0.4249	0.4248	-3.3083	-3.2214	-3.2214	-2.5589
121	0.4247	0.4248	0.4247	-3.3083	-3.3642	-3.3083	-2.7661
123	0.4251	0.4248	0.4248	-3.3811	-3.3642	-3.3563	-2.7023
125	0.4251	0.4251	0.4251	-3.3811	-3.4226	-3.3618	-2.7311
127	0.4249	0.4251	0.4251	-3.5060	-3.4226	-3.4226	-2.7724
129	0.4249	0.4248	0.4249	-3.5060	-3.5768	-3.5060	-2.9878
131	0.4251	0.4248	0.4250	-3.6001	-3.5768	-3.5564	-3.0106
133	0.4251	0.4252	0.4251	-3.6001	-3.6324	-3.5907	-2.9697
135	0.4250	0.4252	0.4251	-3.7044	-3.6324	-3.6324	-3.0098
137	0.4250	0.4249	0.4250	-3.7044	-3.7799	-3.7044	-3.1626
139	0.4250	0.4249	0.4249	-3.8198	-3.7799	-3.7799	-3.2783
141	0.4250	0.4251	0.4250	-3.8198	-3.8487	-3.8021	-3.1614
143	0.4250	0.4251	0.4251	-3.9110	-3.8487	-3.8476	-3.2326
145	0.4250	0.4249	0.4250	-3.9110	-3.9922	-3.9110	-3.4081

147	0.4249	0.4249	0.4249	-4.0473	-3.9922	-3.9859	-3.3907
149	0.4249	0.4250	0.4250	-4.0473	-4.0784	-4.0301	-3.4488
151	0.4250	0.4250	0.4250	-4.1324	-4.0784	-4.0784	-3.4477
153	0.4250	0.4249	0.4250	-4.1324	-4.2127	-4.1324	-3.6682
155	0.4248	0.4249	0.4249	-4.2813	-4.2127	-4.2127	-3.7313
157	0.4248	0.4249	0.4248	-4.2813	-4.3199	-4.2777	-3.6603
159	0.4249	0.4249	0.4249	-4.3629	-4.3199	-4.3045	-3.6772
161	0.4249	0.4248	0.4249	-4.3629	-4.4357	-4.3629	-3.8041
163	0.4247	0.4248	0.4248	-4.5047	-4.4357	-4.4357	-3.8549
165	0.4247	0.4248	0.4248	-4.5047	-4.5372	-4.4901	-3.9048
167	0.4248	0.4248	0.4248	-4.5835	-4.5372	-4.5372	-3.9073
169	0.4248	0.4248	0.4248	-4.5835	-4.6508	-4.5835	-4.0322
171	0.4247	0.4248	0.4248	-4.7208	-4.6508	-4.6508	-4.1224
173	0.4247	0.4247	0.4247	-4.7208	-4.7549	-4.7156	-4.1760
175	0.4248	0.4247	0.4247	-4.8037	-4.7549	-4.7541	-4.1262
177	0.4248	0.4247	0.4248	-4.8037	-4.8616	-4.8037	-4.2657
179	0.4246	0.4247	0.4247	-4.9381	-4.8616	-4.8616	-4.4230
181	0.4246	0.4246	0.4246	-4.9381	-4.9852	-4.9337	-4.3303
183	0.4247	0.4246	0.4246	-5.0312	-4.9852	-4.9743	-4.3497
185	0.4247	0.4247	0.4247	-5.0312	-5.0814	-5.0312	-4.4438
187	0.4246	0.4247	0.4247	-5.1486	-5.0814	-5.0814	-4.5769
189	0.4246	0.4246	0.4246	-5.1486	-5.2065	-5.1486	-4.6221
191	0.4246	0.4246	0.4246	-5.2479	-5.2065	-5.2058	-4.5771
193	0.4246	0.4246	0.4246	-5.2479	-5.2959	-5.2479	-4.6600
195	0.4246	0.4246	0.4246	-5.3601	-5.2959	-5.2959	-4.8366
197	0.4246	0.4245	0.4245	-5.3601	-5.4229	-5.3601	-4.7853
199	0.4245	0.4245	0.4245	-5.4810	-5.4229	-5.4197	-4.8042
201	0.4245	0.4246	0.4245	-5.4810	-5.5240	-5.4810	-4.8838
203	0.4245	0.4246	0.4246	-5.5790	-5.5240	-5.5240	-5.1033
205	0.4245	0.4245	0.4245	-5.5790	-5.6311	-5.5790	-5.0383
207	0.4245	0.4245	0.4245	-5.6927	-5.6311	-5.6311	-5.0267
209	0.4245	0.4246	0.4245	-5.6927	-5.7340	-5.6927	-5.0850
211	0.4246	0.4246	0.4246	-5.7840	-5.7340	-5.7340	-5.3068
213	0.4246	0.4245	0.4246	-5.7840	-5.8422	-5.7840	-5.3066

215	0.4245	0.4245	0.4245	-5.9048	-5.8422	-5.8387	-5.2315
217	0.4245	0.4245	0.4245	-5.9048	-5.9540	-5.9036	-5.2986
219	0.4246	0.4245	0.4245	-6.0026	-5.9540	-5.9540	-5.5141
221	0.4246	0.4245	0.4246	-6.0026	-6.0654	-6.0026	-5.4987
223	0.4245	0.4245	0.4245	-6.1248	-6.0654	-6.0654	-5.4805

Table E.2. Performance summary for I and Q filters where *TBW*=0.150.

M	$F_{p.Q}$	$F_{p.I}$	$F_{p.IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.3273	0.3426	0.3426	-1.4334	-1.5094	-1.2215	0.1065
23	0.3515	0.3426	0.3515	-1.3934	-1.5094	-1.3738	-0.4649
25	0.3515	0.3458	0.3515	-1.3934	-1.4839	-1.3813	-0.5828
27	0.3361	0.3458	0.3458	-1.7554	-1.4839	-1.4839	-0.3503
29	0.3361	0.3341	0.3361	-1.7554	-2.0320	-1.7347	-0.4601
31	0.3430	0.3341	0.3341	-1.9619	-2.0320	-1.9339	-0.7796
33	0.3430	0.3454	0.3435	-1.9619	-1.9516	-1.9480	-1.4480
35	0.3428	0.3454	0.3428	-2.0878	-1.9516	-1.9478	-1.0011
37	0.3428	0.3402	0.3402	-2.0878	-2.3139	-2.0755	-0.9814
39	0.3430	0.3402	0.3424	-2.3995	-2.3139	-2.2372	-1.1704
41	0.3430	0.3461	0.3438	-2.3995	-2.3892	-2.3686	-1.6299
43	0.3468	0.3461	0.3461	-2.4438	-2.3892	-2.3733	-1.5982
45	0.3468	0.3459	0.3468	-2.4438	-2.5866	-2.4438	-1.5386
47	0.3461	0.3459	0.3459	-2.7262	-2.5866	-2.5866	-1.6466
49	0.3461	0.3479	0.3461	-2.7262	-2.7766	-2.7262	-1.8741
51	0.3492	0.3479	0.3479	-2.8180	-2.7766	-2.7747	-2.1839
53	0.3492	0.3493	0.3492	-2.8180	-2.9138	-2.8180	-2.0603
55	0.3490	0.3493	0.3490	-3.0445	-2.9138	-2.9138	-2.0272
57	0.3490	0.3494	0.3491	-3.0445	-3.1488	-3.0365	-2.1714
59	0.3501	0.3494	0.3494	-3.2189	-3.1488	-3.1488	-2.4940
61	0.3501	0.3505	0.3501	-3.2189	-3.3003	-3.2189	-2.5383
63	0.3504	0.3505	0.3505	-3.4128	-3.3003	-3.3003	-2.5280
65	0.3504	0.3502	0.3504	-3.4128	-3.5346	-3.4128	-2.6134
67	0.3503	0.3502	0.3502	-3.6423	-3.5346	-3.5346	-2.8535
69	0.3503	0.3505	0.3503	-3.6423	-3.7345	-3.6423	-2.9972
71	0.3505	0.3505	0.3505	-3.8362	-3.7345	-3.7345	-2.9920
73	0.3505	0.3504	0.3505	-3.8362	-3.9548	-3.8362	-3.0341
75	0.3502	0.3504	0.3504	-4.0800	-3.9548	-3.9548	-3.1685
77	0.3502	0.3501	0.3502	-4.0800	-4.1963	-4.0800	-3.4303
79	0.3501	0.3501	0.3501	-4.3019	-4.1963	-4.1963	-3.4633
81	0.3501	0.3501	0.3501	-4.3019	-4.4078	-4.2985	-3.4579
83	0.3500	0.3501	0.3501	-4.5279	-4.4078	-4.4072	-3.5363

85	0.3500	0.3498	0.3500	-4.5279	-4.6591	-4.5279	-3.7486
87	0.3497	0.3498	0.3498	-4.7493	-4.6591	-4.6591	-3.9277
89	0.3497	0.3497	0.3497	-4.7493	-4.8428	-4.7387	-3.8954
91	0.3498	0.3497	0.3498	-4.9664	-4.8428	-4.8357	-3.9217
93	0.3498	0.3497	0.3498	-4.9664	-5.0898	-4.9664	-4.1018
95	0.3496	0.3497	0.3497	-5.2034	-5.0898	-5.0898	-4.3575
97	0.3496	0.3496	0.3496	-5.2034	-5.2893	-5.1879	-4.3382
99	0.3496	0.3496	0.3496	-5.4059	-5.2893	-5.2758	-4.3122
101	0.3496	0.3497	0.3496	-5.4059	-5.5096	-5.4000	-4.4658
103	0.3497	0.3497	0.3497	-5.6086	-5.5096	-5.5096	-4.8256
105	0.3497	0.3497	0.3497	-5.6086	-5.7019	-5.5824	-4.7343
107	0.3497	0.3497	0.3497	-5.8312	-5.7019	-5.6835	-4.7081
109	0.3497	0.3497	0.3497	-5.8312	-5.9181	-5.8135	-4.8395
111	0.3498	0.3497	0.3497	-6.0043	-5.9181	-5.9181	-5.1758
113	0.3498	0.3498	0.3498	-6.0043	-6.1003	-6.0043	-5.1577
115	0.3497	0.3498	0.3498	-6.2211	-6.1003	-6.1003	-5.1148
117	0.3497	0.3497	0.3497	-6.2211	-6.3406	-6.2211	-5.2217
119	0.3498	0.3497	0.3497	-6.4164	-6.3406	-6.3406	-5.5242
121	0.3498	0.3498	0.3498	-6.4164	-6.5168	-6.4164	-5.6075
123	0.3497	0.3498	0.3497	-6.6462	-6.5168	-6.4953	-5.5261
125	0.3497	0.3497	0.3497	-6.6462	-6.7763	-6.6003	-5.6122
127	0.3497	0.3497	0.3497	-6.8688	-6.7763	-6.7609	-5.8882
129	0.3497	0.3498	0.3497	-6.8688	-6.9543	-6.8628	-6.0179
131	0.3497	0.3498	0.3498	-7.0704	-6.9543	-6.9084	-5.9392
133	0.3497	0.3496	0.3497	-7.0704	-7.2131	-7.0544	-6.0092
135	0.3496	0.3496	0.3496	-7.2748	-7.2131	-7.2039	-6.2605

APPENDIX F - PERFORMANCE DATA FOR QUADRATURE DEMODULATION FILTERS DERIVED FROM PROTOTYPE FILTERS DESIGNED USING THE KAISER WINDOW

Table F.1. Performance summary for I and Q filters where *TBW*=0.075.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.3787	0.4159	0.3983	-1.1550	-0.8244	-0.8244	0.2559
23	0.4208	0.4159	0.4208	-0.7272	-0.8414	-0.7272	-0.1626
25	0.4208	0.4159	0.4208	-0.7272	-0.8414	-0.7272	-0.1626
27	0.4121	0.4113	0.4113	-1.1069	-0.8694	-0.8694	0.1001
29	0.4079	0.4251	0.4147	-1.1330	-0.9649	-0.9649	0.0348
31	0.4257	0.4218	0.4249	-0.9002	-0.9967	-0.9002	-0.4481
33	0.4235	0.4195	0.4226	-0.9298	-1.0265	-0.9298	-0.4751
35	0.4155	0.4179	0.4179	-1.2708	-1.0537	-1.0537	-0.2071
37	0.4142	0.4225	0.4164	-1.2954	-1.2448	-1.2448	-0.2904
39	0.4248	0.4216	0.4235	-1.1821	-1.2628	-1.1821	-0.7548
41	0.4244	0.4212	0.4231	-1.1884	-1.2692	-1.1884	-0.7587
43	0.4175	0.4211	0.4211	-1.4709	-1.2673	-1.2673	-0.4920
45	0.4177	0.4209	0.4180	-1.4655	-1.5008	-1.4578	-0.5744
47	0.4238	0.4214	0.4226	-1.4278	-1.4906	-1.4278	-1.0269
49	0.4245	0.4221	0.4233	-1.4158	-1.4801	-1.4158	-1.0123
51	0.4191	0.4228	0.4219	-1.6548	-1.4689	-1.4689	-0.7605
53	0.4200	0.4203	0.4200	-1.6456	-1.7412	-1.6456	-0.8419
55	0.4230	0.4214	0.4222	-1.6824	-1.7243	-1.6824	-1.3013
57	0.4243	0.4226	0.4234	-1.6584	-1.7038	-1.6584	-1.2647
59	0.4205	0.4239	0.4225	-1.8429	-1.6811	-1.6811	-1.0131
61	0.4216	0.4204	0.4214	-1.8269	-1.9567	-1.8269	-1.0968
63	0.4227	0.4218	0.4222	-1.9183	-1.9370	-1.9159	-1.5701
65	0.4242	0.4231	0.4236	-1.8904	-1.9148	-1.8903	-1.5203
67	0.4217	0.4245	0.4232	-2.0233	-1.8909	-1.8909	-1.2542
69	0.4228	0.4211	0.4222	-2.0103	-2.1505	-2.0103	-1.3414
71	0.4227	0.4223	0.4225	-2.1442	-2.1418	-2.1299	-1.8186
73	0.4241	0.4236	0.4239	-2.1231	-2.1266	-2.1102	-1.7533
75	0.4228	0.4249	0.4239	-2.2038	-2.1056	-2.1056	-1.4943
77	0.4239	0.4221	0.4230	-2.1954	-2.3277	-2.1954	-1.5776
79	0.4230	0.4230	0.4230	-2.3527	-2.3345	-2.3316	-2.0594

81	0.4239	0.4238	0.4239	-2.3571	-2.3388	-2.3318	-1.9861
83	0.4235	0.4248	0.4242	-2.3897	-2.3292	-2.3292	-1.7359
85	0.4243	0.4228	0.4236	-2.3930	-2.5048	-2.3930	-1.8162
87	0.4232	0.4234	0.4233	-2.5592	-2.5284	-2.5284	-2.3054
89	0.4240	0.4241	0.4240	-2.5803	-2.5446	-2.5446	-2.2105
91	0.4242	0.4249	0.4246	-2.5761	-2.5516	-2.5365	-1.9654
93	0.4248	0.4236	0.4242	-2.5901	-2.6762	-2.5901	-2.0541
95	0.4237	0.4240	0.4238	-2.7490	-2.7139	-2.7139	-2.5419
97	0.4240	0.4244	0.4241	-2.7952	-2.7491	-2.7491	-2.4330
99	0.4246	0.4249	0.4248	-2.7677	-2.7787	-2.7436	-2.1933
101	0.4250	0.4242	0.4246	-2.7958	-2.8504	-2.7958	-2.2843
103	0.4241	0.4244	0.4242	-2.9369	-2.9013	-2.9013	-2.7789
105	0.4241	0.4245	0.4243	-3.0039	-2.9529	-2.9529	-2.6540
107	0.4249	0.4248	0.4248	-2.9643	-3.0066	-2.9566	-2.4203
109	0.4251	0.4246	0.4249	-3.0095	-3.0312	-2.9935	-2.5142
111	0.4244	0.4247	0.4245	-3.1218	-3.0928	-3.0928	-3.0201
113	0.4243	0.4247	0.4245	-3.2087	-3.1605	-3.1605	-2.8839
115	0.4250	0.4247	0.4247	-3.1694	-3.2328	-3.1694	-2.6471
117	0.4251	0.4249	0.4251	-3.2252	-3.2214	-3.1968	-2.7427
119	0.4247	0.4249	0.4248	-3.3082	-3.2859	-3.2859	-3.2598
121	0.4245	0.4248	0.4246	-3.4011	-3.3642	-3.3642	-3.1038
123	0.4251	0.4246	0.4248	-3.3811	-3.4571	-3.3811	-2.8713
125	0.4250	0.4251	0.4250	-3.4546	-3.4226	-3.4136	-2.9711
127	0.4249	0.4250	0.4250	-3.5060	-3.4938	-3.4917	-3.4945
129	0.4246	0.4248	0.4248	-3.5983	-3.5767	-3.5713	-3.3229
131	0.4251	0.4246	0.4248	-3.6001	-3.6800	-3.6001	-3.1012
133	0.4249	0.4252	0.4250	-3.6774	-3.6324	-3.6324	-3.1998
135	0.4250	0.4251	0.4251	-3.7044	-3.6982	-3.6923	-3.7310
137	0.4248	0.4249	0.4249	-3.7889	-3.7799	-3.7674	-3.5417
139	0.4250	0.4246	0.4248	-3.8198	-3.8844	-3.8198	-3.3254
141	0.4248	0.4251	0.4249	-3.9031	-3.8487	-3.8487	-3.4261
143	0.4250	0.4250	0.4250	-3.9110	-3.9133	-3.9028	-3.9646
145	0.4249	0.4249	0.4249	-3.9852	-3.9922	-3.9710	-3.7615
147	0.4249	0.4246	0.4248	-4.0473	-4.0908	-4.0412	-3.5505

149	0.4247	0.4250	0.4249	-4.1326	-4.0784	-4.0784	-3.6581
151	0.4250	0.4250	0.4250	-4.1324	-4.1409	-4.1281	-4.2004
153	0.4249	0.4249	0.4249	-4.1937	-4.2127	-4.1875	-3.9791
155	0.4248	0.4247	0.4248	-4.2813	-4.2968	-4.2540	-3.7722
157	0.4247	0.4249	0.4248	-4.3654	-4.3199	-4.3199	-3.8841
159	0.4249	0.4249	0.4249	-4.3629	-4.3754	-4.3624	-4.4354
161	0.4249	0.4248	0.4248	-4.4087	-4.4357	-4.4087	-4.1966
163	0.4247	0.4247	0.4247	-4.5047	-4.5015	-4.4686	-3.9958
165	0.4246	0.4248	0.4248	-4.5836	-4.5372	-4.5372	-4.1103
167	0.4248	0.4248	0.4248	-4.5835	-4.6004	-4.5835	-4.6771
169	0.4249	0.4248	0.4248	-4.6188	-4.6508	-4.6188	-4.4206
171	0.4247	0.4247	0.4247	-4.7208	-4.6967	-4.6761	-4.2184
173	0.4246	0.4247	0.4247	-4.7847	-4.7549	-4.7460	-4.3368
175	0.4248	0.4247	0.4247	-4.8037	-4.8135	-4.8037	-4.9138
177	0.4248	0.4247	0.4248	-4.8312	-4.8616	-4.8312	-4.6390
179	0.4246	0.4248	0.4246	-4.9381	-4.8916	-4.8916	-4.4406
181	0.4246	0.4246	0.4246	-4.9909	-4.9852	-4.9646	-4.5625
183	0.4247	0.4246	0.4246	-5.0312	-5.0392	-5.0294	-5.1494
185	0.4247	0.4247	0.4247	-5.0588	-5.0814	-5.0584	-4.8556
187	0.4246	0.4247	0.4246	-5.1486	-5.1021	-5.1021	-4.6689
189	0.4246	0.4246	0.4246	-5.1886	-5.2065	-5.1721	-4.7890
191	0.4246	0.4246	0.4246	-5.2479	-5.2599	-5.2479	-5.3861
193	0.4247	0.4246	0.4247	-5.2828	-5.2959	-5.2711	-5.0710
195	0.4246	0.4247	0.4247	-5.3601	-5.3134	-5.3134	-4.8918
197	0.4246	0.4245	0.4245	-5.3928	-5.4229	-5.3888	-5.0169
199	0.4245	0.4245	0.4245	-5.4810	-5.4797	-5.4735	-5.6244
201	0.4246	0.4246	0.4246	-5.5096	-5.5240	-5.5015	-5.2845
203	0.4245	0.4247	0.4246	-5.5790	-5.5466	-5.5416	-5.1134
205	0.4246	0.4245	0.4245	-5.6081	-5.6311	-5.6081	-5.2428
207	0.4245	0.4245	0.4245	-5.6927	-5.6830	-5.6830	-5.8572
209	0.4246	0.4246	0.4246	-5.7233	-5.7340	-5.7172	-5.5030
211	0.4246	0.4246	0.4246	-5.7840	-5.7658	-5.7431	-5.3321
213	0.4246	0.4245	0.4246	-5.8156	-5.8422	-5.8156	-5.4713
215	0.4245	0.4245	0.4245	-5.9048	-5.8957	-5.8952	-6.0950

217	0.4245	0.4245	0.4245	-5.9444	-5.9540	-5.9444	-5.7162
219	0.4246	0.4246	0.4246	-6.0026	-5.9852	-5.9673	-5.5582
221	0.4246	0.4245	0.4246	-6.0416	-6.0654	-6.0322	-5.6958
223	0.4245	0.4245	0.4245	-6.1248	-6.1195	-6.1167	-6.3360

F.2. Performance summary for I and Q filters where *TBW*=0.150.

M	$F_{p,Q}$	$F_{p,I}$	$F_{p,IQ}$	$\log_{10}(2\delta_Q)$	$\log_{10}(2\delta_I)$	$\log_{10}(2\delta_{IQ})$	$\log_{10}(\varphi_e)$
21	0.3248	0.3426	0.3261	-1.4520	-1.5047	-1.4357	-0.1945
23	0.3515	0.3432	0.3475	-1.3922	-1.5077	-1.3922	-0.7695
25	0.3542	0.3458	0.3502	-1.3666	-1.4852	-1.3666	-0.7416
27	0.3361	0.3493	0.3458	-1.7573	-1.4505	-1.4505	-0.5820
29	0.3394	0.3341	0.3394	-1.7411	-2.0333	-1.7411	-0.7514
31	0.3430	0.3395	0.3411	-1.9622	-2.0031	-1.9604	-1.3670
33	0.3498	0.3454	0.3475	-1.8901	-1.9512	-1.8901	-1.2563
35	0.3428	0.3512	0.3473	-2.0872	-1.8904	-1.8904	-1.0833
37	0.3468	0.3402	0.3442	-2.0723	-2.3135	-2.0723	-1.2618
39	0.3430	0.3428	0.3429	-2.3995	-2.3693	-2.3664	-1.9056
41	0.3469	0.3461	0.3467	-2.4136	-2.3894	-2.3669	-1.7253
43	0.3468	0.3508	0.3489	-2.4440	-2.3629	-2.3629	-1.5824
45	0.3499	0.3459	0.3480	-2.4498	-2.5867	-2.4498	-1.7578
47	0.3461	0.3467	0.3464	-2.7263	-2.6833	-2.6833	-2.4426
49	0.3466	0.3479	0.3470	-2.8646	-2.7765	-2.7765	-2.1756
51	0.3492	0.3498	0.3498	-2.8179	-2.8466	-2.7862	-2.0556
53	0.3509	0.3493	0.3502	-2.8657	-2.9138	-2.8605	-2.2530
55	0.3490	0.3495	0.3492	-3.0445	-3.0156	-3.0156	-2.9768
57	0.3479	0.3494	0.3486	-3.2368	-3.1488	-3.1488	-2.6257
59	0.3501	0.3490	0.3493	-3.2189	-3.3217	-3.2189	-2.5271
61	0.3506	0.3505	0.3506	-3.3213	-3.3003	-3.2788	-2.7416
63	0.3504	0.3506	0.3505	-3.4128	-3.3973	-3.3973	-3.5514
65	0.3494	0.3502	0.3499	-3.5795	-3.5346	-3.5346	-3.0850
67	0.3503	0.3489	0.3497	-3.6423	-3.7495	-3.6423	-3.0072
69	0.3500	0.3505	0.3501	-3.7948	-3.7345	-3.7345	-3.2329
71	0.3505	0.3507	0.3506	-3.8362	-3.8297	-3.8284	-4.0957
73	0.3502	0.3504	0.3504	-3.9512	-3.9548	-3.9257	-3.5309
75	0.3502	0.3495	0.3500	-4.0800	-4.1369	-4.0733	-3.4794
77	0.3495	0.3501	0.3498	-4.2532	-4.1963	-4.1963	-3.7348
79	0.3501	0.3502	0.3502	-4.3019	-4.2954	-4.2953	-4.5875
81	0.3503	0.3501	0.3501	-4.3712	-4.4078	-4.3688	-3.9758
83	0.3500	0.3499	0.3500	-4.5279	-4.5194	-4.4828	-3.9542

85	0.3494	0.3498	0.3497	-4.6895	-4.6591	-4.6484	-4.2364
87	0.3497	0.3497	0.3497	-4.7493	-4.7361	-4.7361	-4.9792
89	0.3500	0.3497	0.3499	-4.8304	-4.8428	-4.8191	-4.4338
91	0.3498	0.3500	0.3498	-4.9664	-4.9296	-4.9254	-4.4305
93	0.3497	0.3497	0.3497	-5.0854	-5.0898	-5.0621	-4.7453
95	0.3496	0.3496	0.3496	-5.2034	-5.1940	-5.1940	-5.3152
97	0.3497	0.3496	0.3497	-5.2655	-5.2893	-5.2655	-4.8802
99	0.3496	0.3499	0.3497	-5.4059	-5.3738	-5.3738	-4.9181
101	0.3498	0.3497	0.3497	-5.4889	-5.5096	-5.4846	-5.2613
103	0.3497	0.3497	0.3497	-5.6086	-5.6084	-5.5989	-5.6709
105	0.3496	0.3497	0.3496	-5.7189	-5.7019	-5.7019	-5.3264
107	0.3497	0.3497	0.3497	-5.8312	-5.8182	-5.8081	-5.4012
109	0.3498	0.3497	0.3498	-5.8956	-5.9181	-5.8956	-5.7977
111	0.3498	0.3497	0.3498	-6.0043	-6.0185	-6.0043	-6.0189
113	0.3497	0.3498	0.3497	-6.1203	-6.1003	-6.0899	-5.7726
115	0.3497	0.3497	0.3497	-6.2211	-6.2509	-6.2204	-5.8882
117	0.3498	0.3497	0.3497	-6.3261	-6.3406	-6.3261	-6.3154
119	0.3498	0.3498	0.3498	-6.4164	-6.4210	-6.4049	-6.3704
121	0.3497	0.3498	0.3498	-6.5441	-6.5168	-6.5168	-6.2337
123	0.3497	0.3497	0.3497	-6.6462	-6.6687	-6.6416	-6.3721
125	0.3497	0.3497	0.3497	-6.7694	-6.7763	-6.7616	-6.7826
127	0.3497	0.3497	0.3497	-6.8688	-6.8587	-6.8486	-6.7337
129	0.3497	0.3498	0.3498	-6.9670	-6.9543	-6.9392	-6.6820
131	0.3497	0.3497	0.3497	-7.0704	-7.0903	-7.0704	-6.8766
133	0.3496	0.3496	0.3496	-7.2054	-7.2131	-7.1981	-7.1759
135	0.3496	0.3497	0.3496	-7.2748	-7.2916	-7.2700	-7.1251

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ON THE APPLICATION OF THE PARKS-MCCLELLAN ALGORITHM TO THE DESIGN OF QUADRATURE DEMODULATION FILTERSI (U)

4. AUTHORS (Last name, first name, middle initial)

INKOL, ROBERT J. AND PHUOC, NGUYEN

5. DATE OF PUBLICATION (month and year of publication of document)

JUNE 99

6a. NO. OF PAGES (total containing information. Include Annexes, Appendices, etc.)

73

6b. NO. OF REFS (total cited in document)

12

7. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.

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10a. ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity. This number must be unique to this document.)

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